Project GREAT (2005)

General Relativity Einstein / Essen Anniversary Test

GREAT^2

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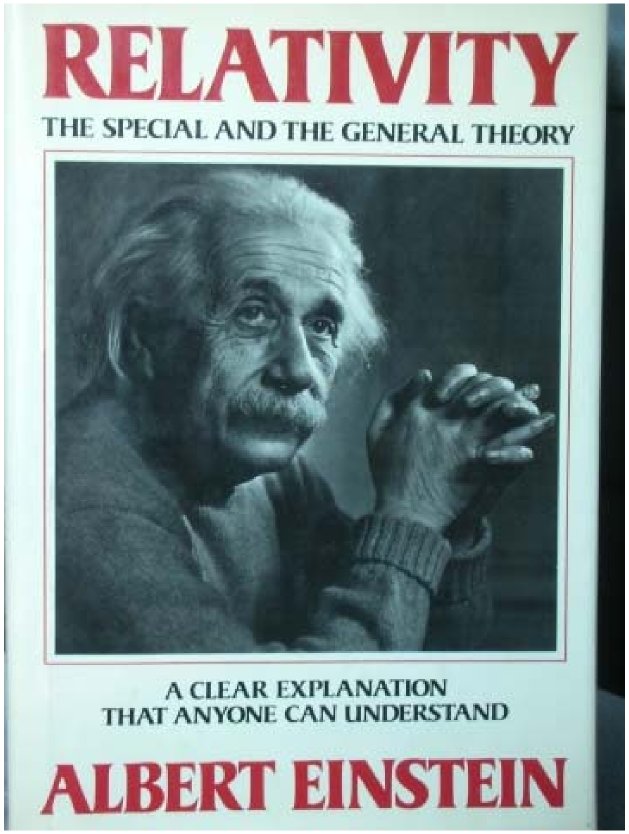
Washington DC

Introduction

- Project GREAT in 2005
  - Attempt to prove the theory of relativity
  - Take cesium clocks up a mountain
  - Do clocks really speed up or slow down?
- Celebrate 100th anniversary of 1905
  - Albert Einstein's "Annus Mirabilis"
- Celebrate 50th anniversary of 1955
  - Louis Essen's NPL cesium clock

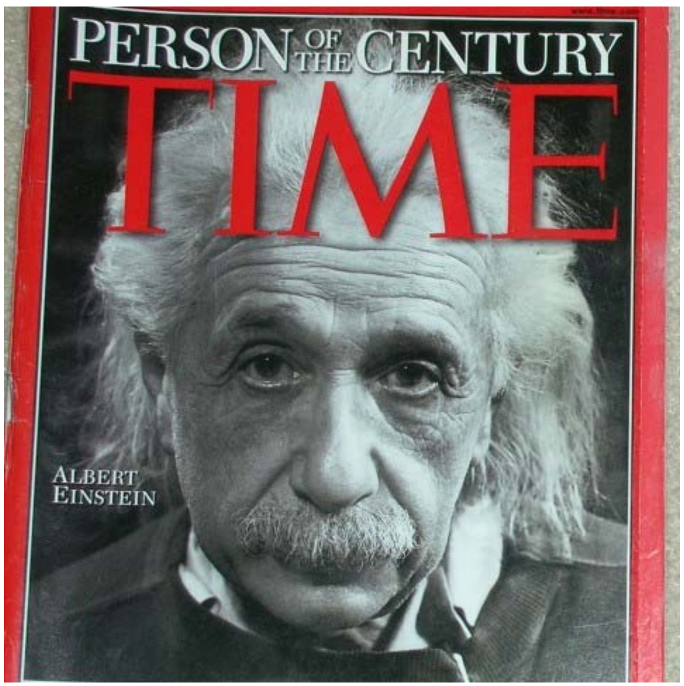
Albert Einstein

- Who was Einstein?
  - Need I say more…
  - Theory of relativity
  - Time is not absolute
  - SR, GR, space-time
  - Bold predictions
  - Later confirmed
  - Enormous influence



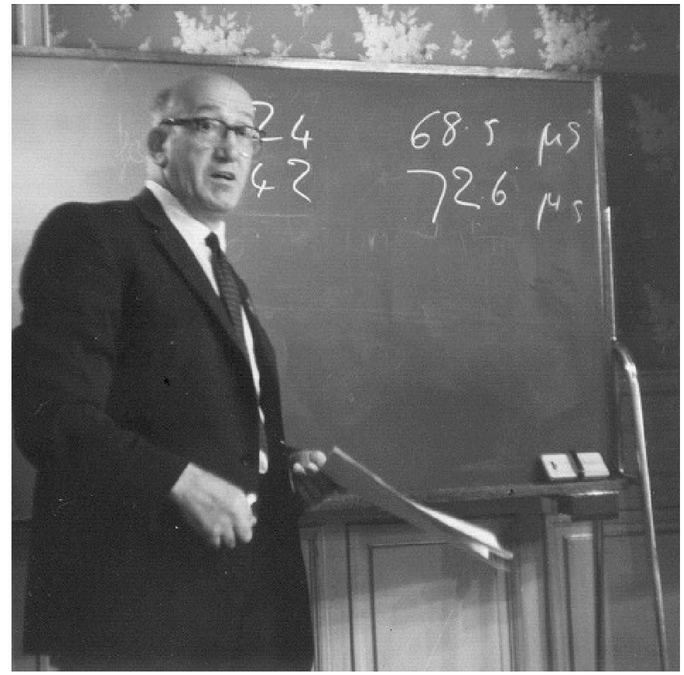
Einstein and 2005

- 100^{th} anniversary of relativity: books, magazines, radio, TV, web sites, "Physics Year", lectures…



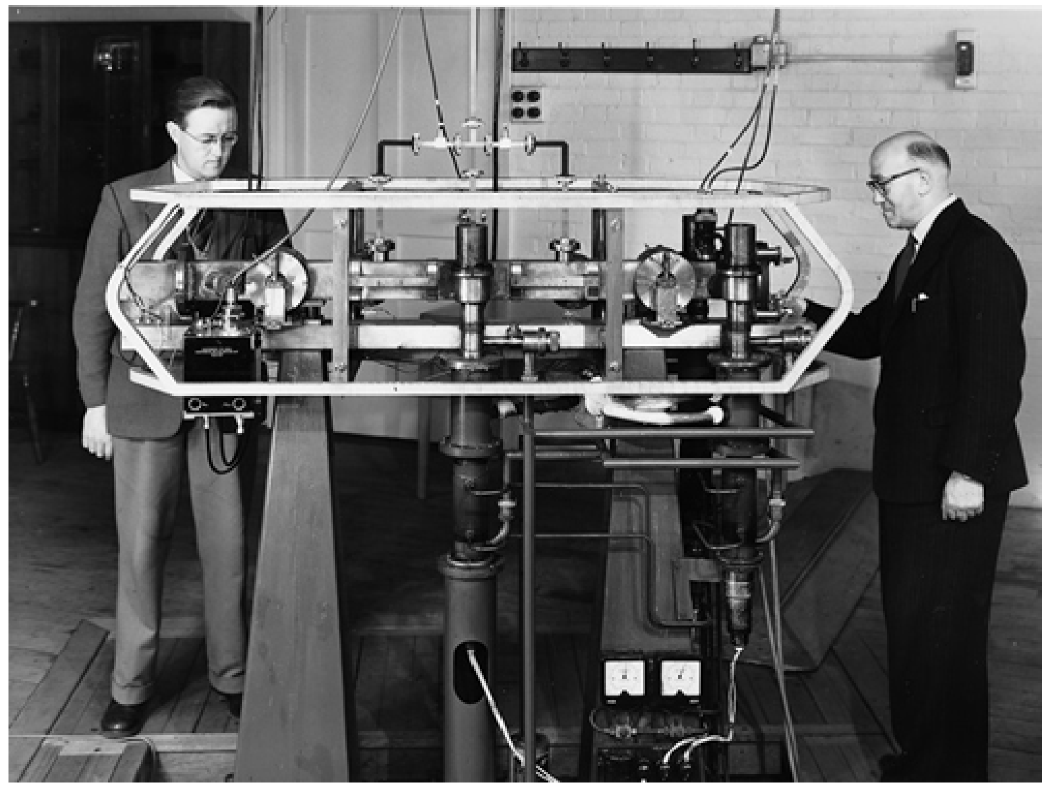
Louis Essen

- Who was Essen?
  - First Cesium Clock
  - Joint NPL USNO project to calibrate atomic time against astronomical time
  - 9 192 631 770 Hz
  - Book: “Famous for a second”



Essen and 2005

- 50th anniversary of atomic time
- NPL Caesium



Jack Parry and Louis Essen
Photo from www.npl.co.uk/essen/

Cs Second

- 1954…1958
- How long is a second?

comparison between the cesium beam at Teddington and the moon camera at Washington. From an analysis of the various factors involved we have adopted a probable error of ±20 cps.

We find, thus, the transition frequency of cesium (4, 0) — (3, 0) at zero magnetic field is

$$\nu_E = 9\,192\,631\,770 \pm 20 \text{ cycles}$$

per second (of E.T.) at 1957.0.

The mean epoch is specified because there is a possibility that the gravitational and atomic time scales may not be the same, and may change secularly. Future determinations of ν_E will decide this question.

VOLUME 1, NUMBER 3 PHYSICAL REVIEW LETTERS AUGUST 1, 1958

Table I. Results for ν_E obtained from four different sets of data.

	Means		ΔH (sec)	$\nu_E - \bar{\nu}_U$	ν_E	$\Delta T''$ (sec/yr²)
1.	ΔT_θ,	1954.25-1958.25	+1.146	-121	9 192 631 761	+ 0.17
2.	ΔT_0,	1955.25-1958.25	1.085	-115	767	+ 0.10
3.	ΔT_C,	1954.25-1958.25	1.035	-110	772	+ 0.12
4.	ΔT_C,	1955.25-1958.25	0.966	-102	780	+ 0.17

Louis Essen

- 10 years later …
- Essen at NPL with a HP 5060A “Flying Clock”

Fig. 10. *Dr. L. Essen, who developed world's first Cesium[133] atomic-controlled frequency standard at National Physical Laboratories, examines compact cesium-beam frequency standard that controls traveling clock.*

Flying Clocks in the 1960's

- Starting in 1964 with HP 5060A
- Portable transistorized cesium clock
- Hundreds of clock trips
- Remote synchronization to μs levels
- See HP Journals: 1964, 65, 66, 67
- 1965 world-wide time synchronization
- Paved the way for flying clock relativity experiments in the 1970's

Relativity and Clocks

- High-level summary:
  - Clocks run slower if they move at high velocity (SR)
  - Clocks run slower in the presence of greater gravity (GR)
  - Clocks lose time traveling East (Sagnac)
- This implies:
  - According to general relativity, stationary clocks on mountains run faster.

And so… 2005

- General
  Relativity
  Einstein
  Essen
  Anniversary
  Test

- Project GRE^{2}AT

Chapter 2

- Flying clock experiments

1971 Hafele & Keating

- PTTI, vol 3, 1971
- Time, Oct 18, 1971
- Science, Jul 14, 1972

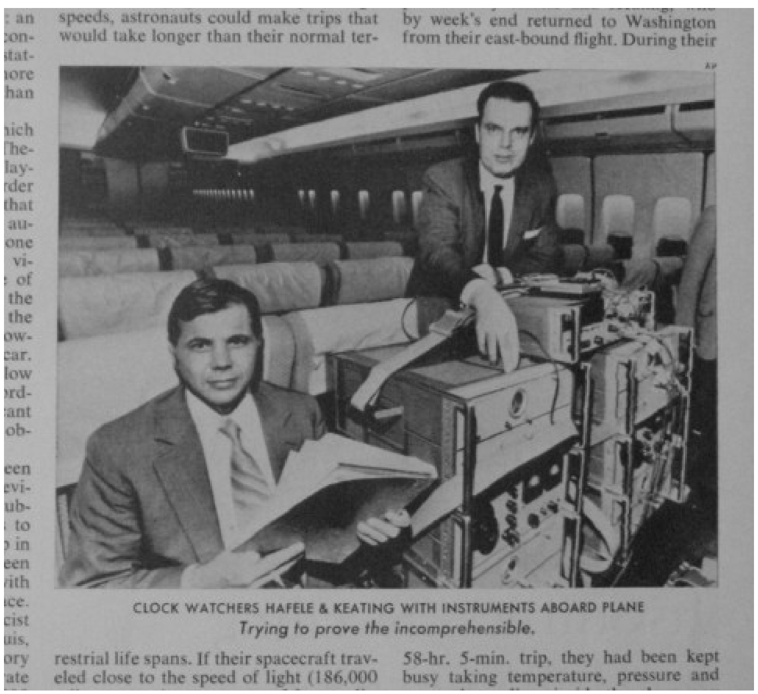


Table 1. Observed relativistic time differences from application of the correlated rate-change method to the time intercomparison data for the flying ensemble. Predicted values are listed for comparison with the mean of the observed values; S.D., standard deviation.

Clock serial No.	$\Delta\tau$ (nsec) Eastward*	Westward
120	− 57	277
361	− 74	284
408	− 55	266
447	− 51	266
Mean ± S.D.	− 59 ± 10	273 ± 7
Predicted ± Error est.	− 40 ± 23	275 ± 21

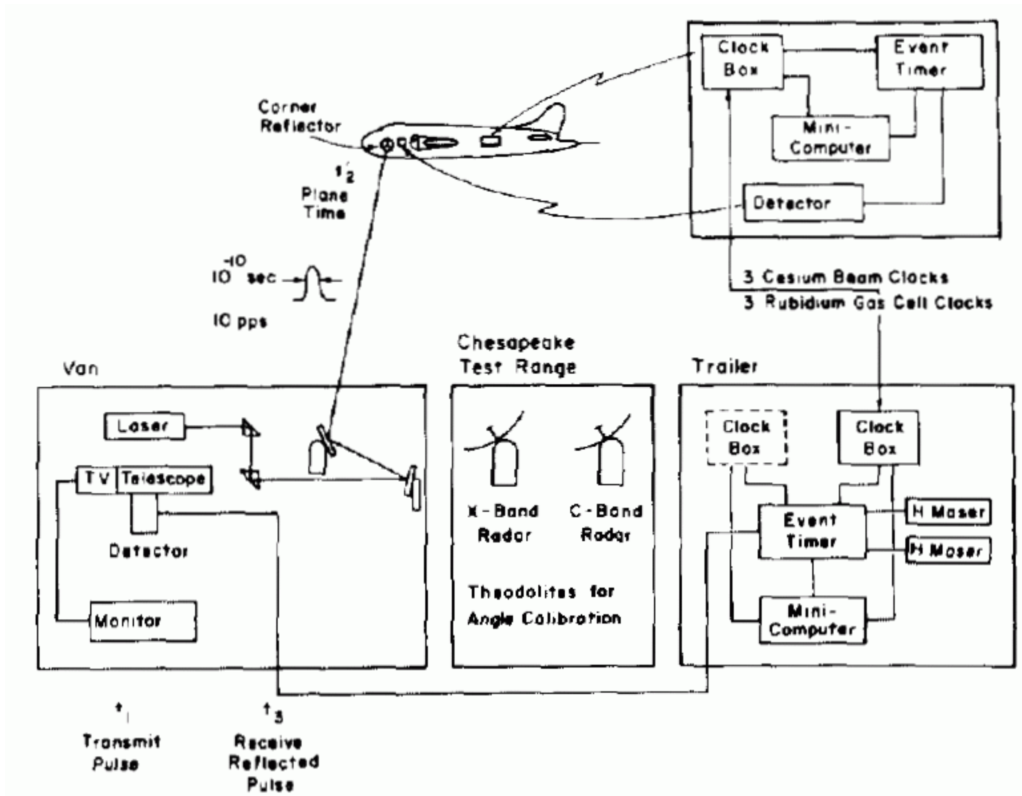
* Negative signs indicate that upon return the time indicated on the flying clocks was less than the time indicated on the MEAN(USNO) clock of the U.S. Naval Observatory.

Hafele & Keating

- Round-the-world (EW/WE) flying clocks
- Hafele thought there would be effect
- Keating didn't (was open to finding out)
- Many scientists were "certain"
- Results were stunning: all 4 clocks showed difference between EW and WE
- Relatively simple, cheap experiment

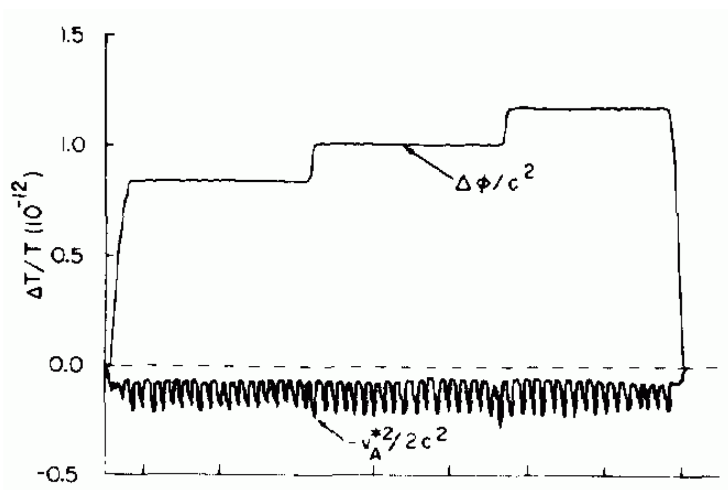
1975 C.O. Alley

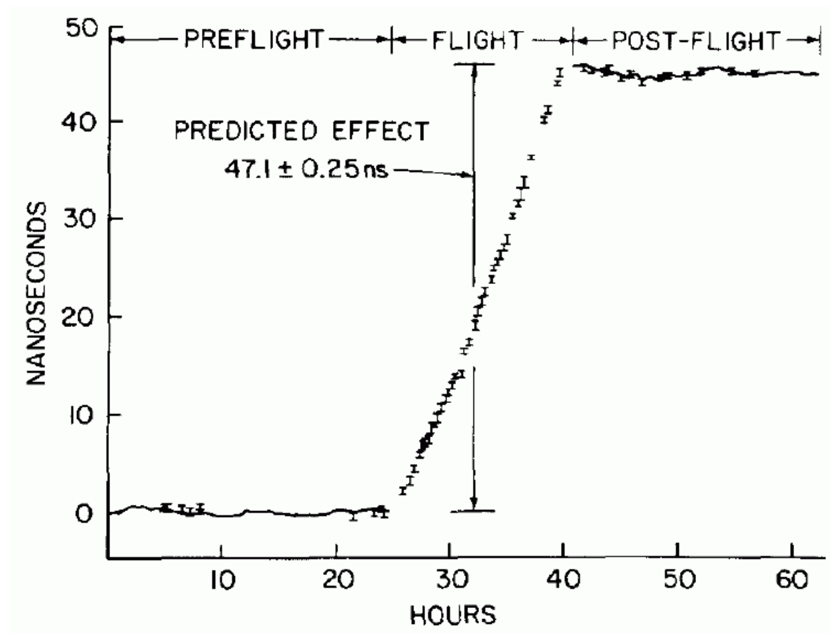
- Multiple flights
- P3C airplane
- Chesapeake Bay
- Environment
- Multiple clocks
  - 3 Cs, 3 Rb, 2 HM
  - L.C. 5061A
- Laser link



C.O. Alley

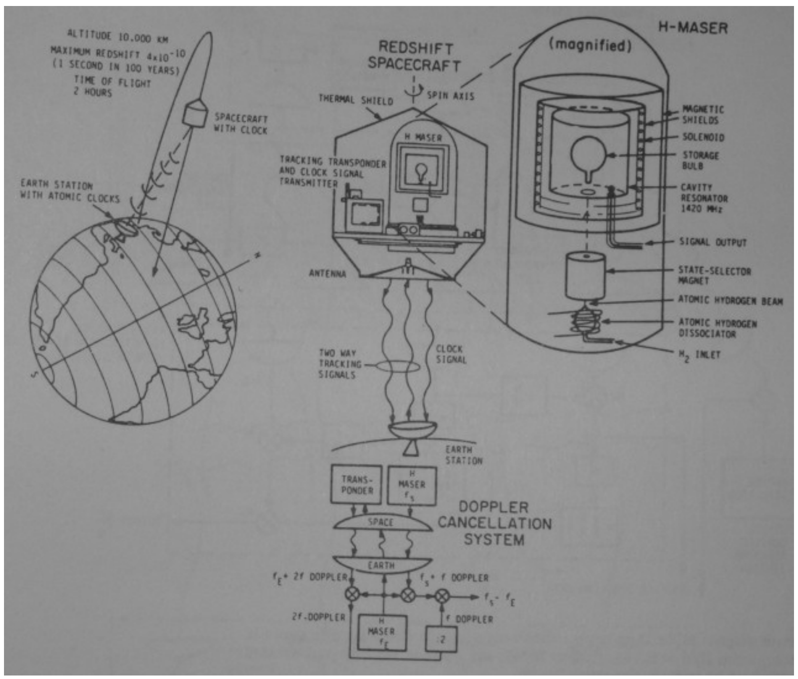
- 3 different altitudes (25, 30, 35 000)
- 5 flights, 2 h, multiple clocks, 0.5%





1976 R.F.C Vessot & M. Levine

- H-maser up, up and away: 10,000 km!
- NASA, ground stations, up/down links
- GP-A amazing results, 0.0070%, 70 ppm



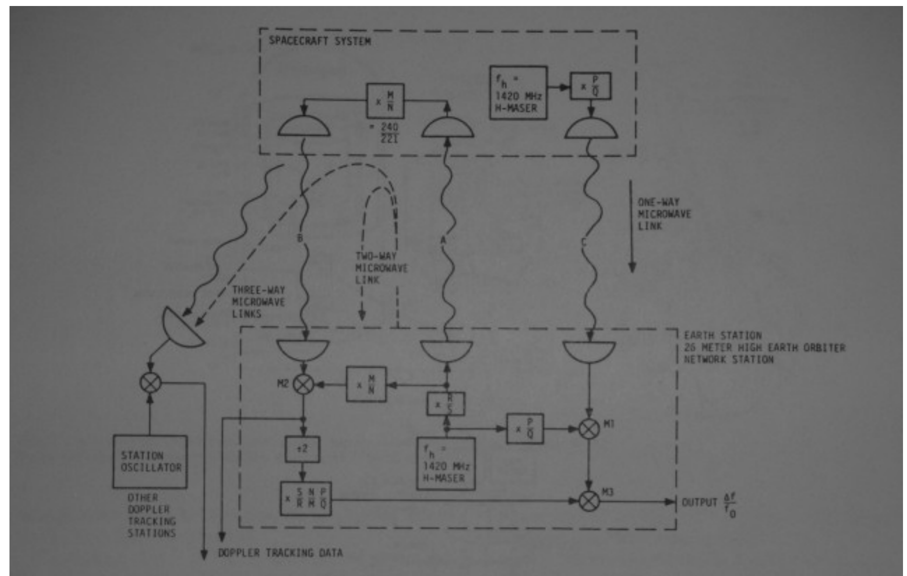
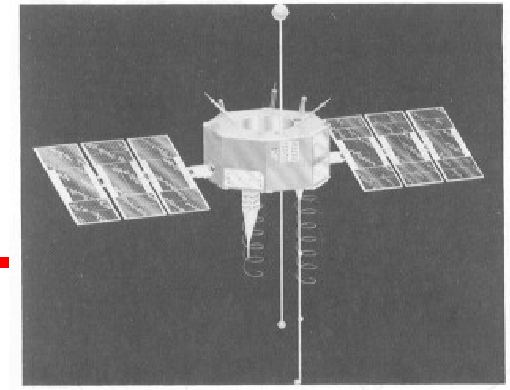


Fig. 5. A schematic diagram of the Doppler cancelling system used to obtain the data shown in Fig. 4. The frequency ratios $R/S = 82/55$, $P/Q = 76/49$, and $N/M = 221/240$ were selected to minimize the effect of ionospheric dispersion.

1977 NTS-2



- Navigation Test Satellite #2
- First NAVSTAR GPS, cesium in space
- Verify relativistic effects

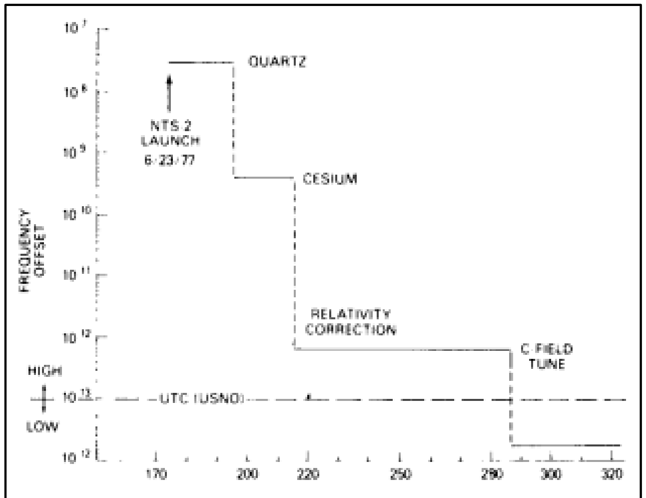


Fig. 27 - Frequency offset summary

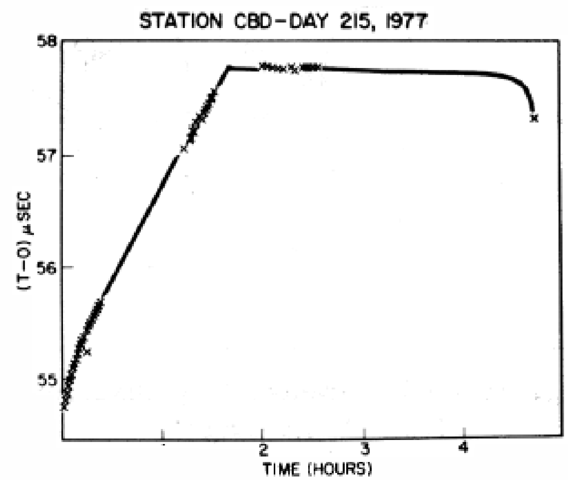


Fig. 21 - Effect of relativity correction

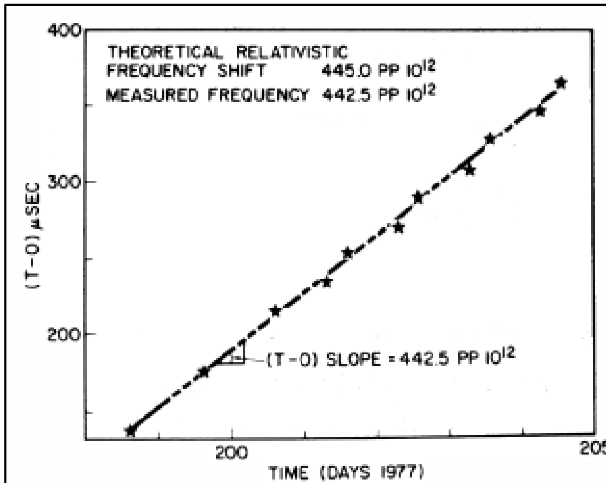


Fig. 20 - Cesium frequency via (T-O) slope

Recent relativity experiments

- 1996 NPL & BBC – 25th anniversary of H&K with London (NPL) to Washington DC (USNO)
- 2000 T. Celano, TSC – military airplane, cesium clocks, comsat modems
- 2002 T. Celano, TSC – similar but with two-way communication, real-time corrections

Recent relativity experiments

- 2000-2002 – CRL (NICT) measured GR time dilation when moving several 5071A between low and high altitude facilities
- Also, many non-atomic clock relativity experiments are in progress
- GP-B, black holes, LIGO, …

Chapter 3

- The Big Idea

Relativity at home?

- Theory of relativity well confirmed
  - With planets, particles, atomic clocks
- But is it so extreme, so exotic, that only places like Harvard, USNO, JPL, or NASA can prove it?
- Or is it possible to perform a home experiment to confirm Einstein's prediction? This seems far-fetched.

Relativity at home

- Methods; take atomic clock
  - At high speed, or
  - To high altitude, or
  - On long eastward or westward trip, or
  - All the above
- Modes; transport using
  - Airplane
  - Rocket
  - Satellite

Relativity at home

- None of those methods work for me:
  - I don't have a plane
  - Rocket or satellite is out of the question
- But I do have:
  - Many atomic clocks
  - Nearby mountains
- So, use a mountain?

Northwest Mountains

- (N) Mt Baker, Glacier Peak
- (S) Mt Rainer, Mt Adams, Mt St Helens
- (E) Olympic range, Cascade range…

Back of Envelope Calculation

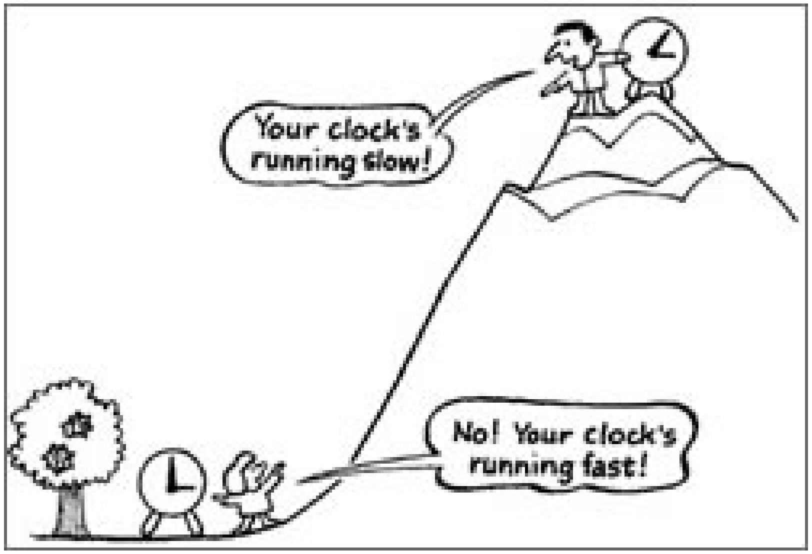
- According to GR, clock frequency changes approximately by:

  $\approx g \cdot \Delta h / c^2$

- On earth, this is

  $\approx 1.09 \times 10^{-16}$/meter

- That's really small!
- Too small for me (or anyone) to measure



From NPL website

1.1×10^{-16} is too small, but

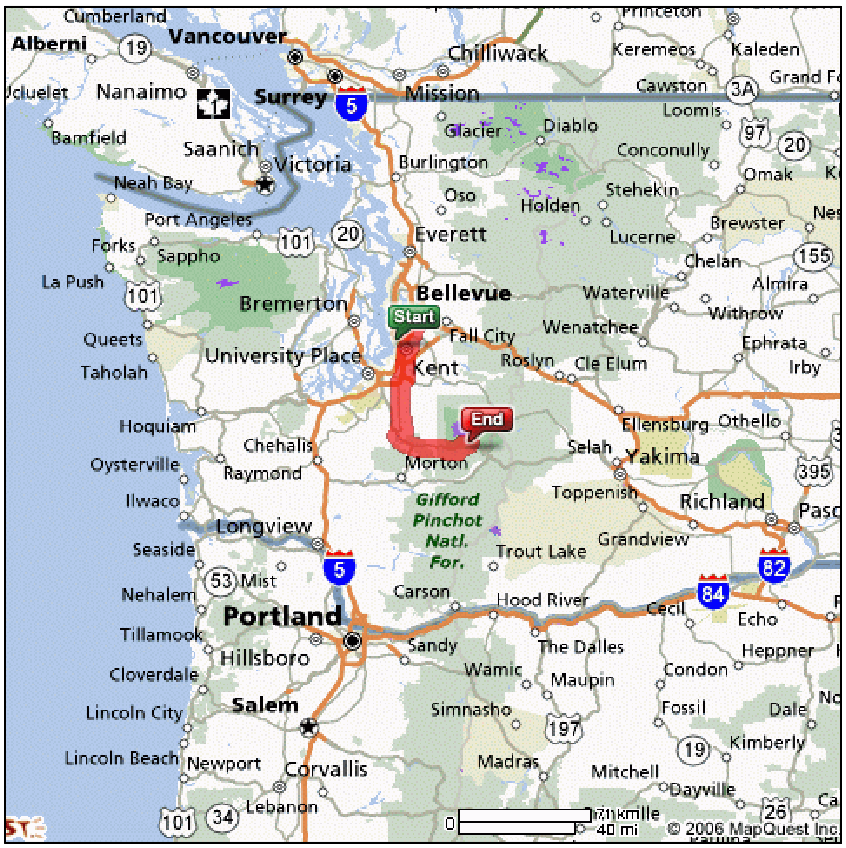
- Say, you go up 1 km instead of 1 m
  $\Delta f = 1.1\times10^{-13} = 0.11$ ps/s
- And stay a whole day
  $\Delta T = \Delta f \times 86400$ s $= 9.5$ ns
- 9 ns is "huge"; so this looks possible!
- The key to detecting time dilation: *go high and stay long*
- Sign is + (blue shift)

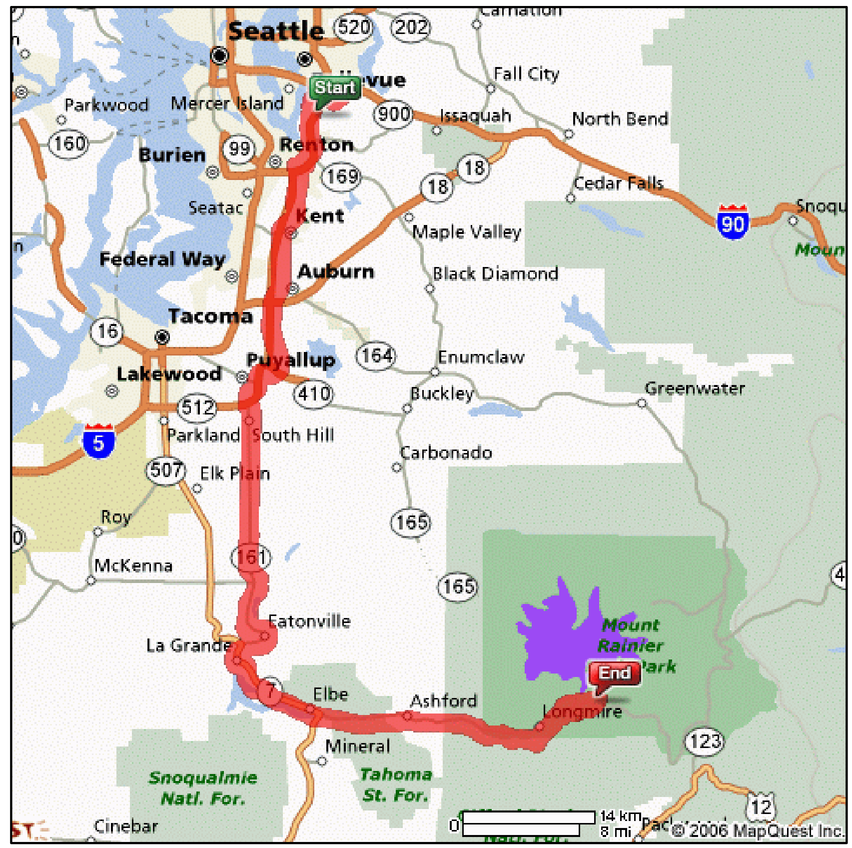
The Big Idea

- Take our 3 kids and 3 cesium clocks up Mt Rainier
- See if Einstein was right about gravity and time
- See if clocks really run faster up there

Map: Seattle to Mt Rainier

- Just 100 miles away (~2½ hours)





Math Detail

$$d\tau' = \frac{d\tau}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- To a first approximation, small v, Δh
- Kinematic: $\Delta f_k \approx -\frac{1}{2} v^2/c^2$
- Gravitation: $\Delta f_g \approx +g\Delta h/c^2$
- Sagnac: $\Delta f_s \approx -\omega R^2 \cos^2(\phi) \cdot \lambda \,/\, c^2$
- Net $\Delta f = \Delta f_k + \Delta f_g + \Delta f_s$
- Total $\Delta T = \sum \Delta f \times T$

Prediction: Sagnac Effect

- $\cos^2$ factor is 0.5 for 45° vs. equator
- 200 ns for 40 000 km round-the-world
  - So 0.001x that for 40 km, or 200 ps
- No effect for N-S travel
- And no effect for same-path round-trip
- So ignore Sagnac effect

Prediction: Velocity Effect

- Automobile speeds are relatively low
  - 65 mph ≈ 96 fps ≈ 100 kph ≈ 30 m/s
- Actual trip is 100 miles and 2.5 hours
  - Average speed below 40 mph
- Worst case 30 m/s for 10^4 s
  - so $\Delta f = 5 \times 10^{-15}$, and $\Delta T = -50$ ps.
- So ignore Velocity factor too!

Prediction: Gravitational

- Guess 40 to 48 hours
- Guess 1640 - 300 = 1340 meters
- $\Delta f = 1.5 \times 10^{-13}$
  - This Δf is very measurable with 5071A
- 1340 m × 40 hours
- $\Delta T \approx +20$ ns
  - This ΔT is very measurable with 53132A

Time Dilation Examples

Transport	Speed (km/h)	Altitude (km)	Δf	ΔT
Walking	5	0	~0	~0 ps/h
Car	100	0	-4×10^{-15}	16 ps/h 0.4 ns/d
Balloon	0	10	$+1.1\times10^{-12}$	+95 ns/d
Plane	900	10	-3.5×10^{-13} $+1.1\times10^{-12}$	-30 ns/d +95 ns/d
GREAT 2+2 h drive	100	0		60 ps
GREAT 40 h stay	0	1.340	$+1.5\times10^{-13}$	20 ns

Summary – Calculations

- “Many, High, Long“
- 3 best clocks (5071A/001)
- 1340 m (5400' – 1000') altitude
- Weekend (40 h)
- 1×10^{-16} & 1340 m & 40h & $\sqrt{3}$
- Estimate ~20 ns
- Estimated accuracy ~2 ns, 10%
- Time dilating at about 500 ps / hour

Clocks on Mountains

- Climb or drive?
- Plus batteries…

Clocks on Mountains

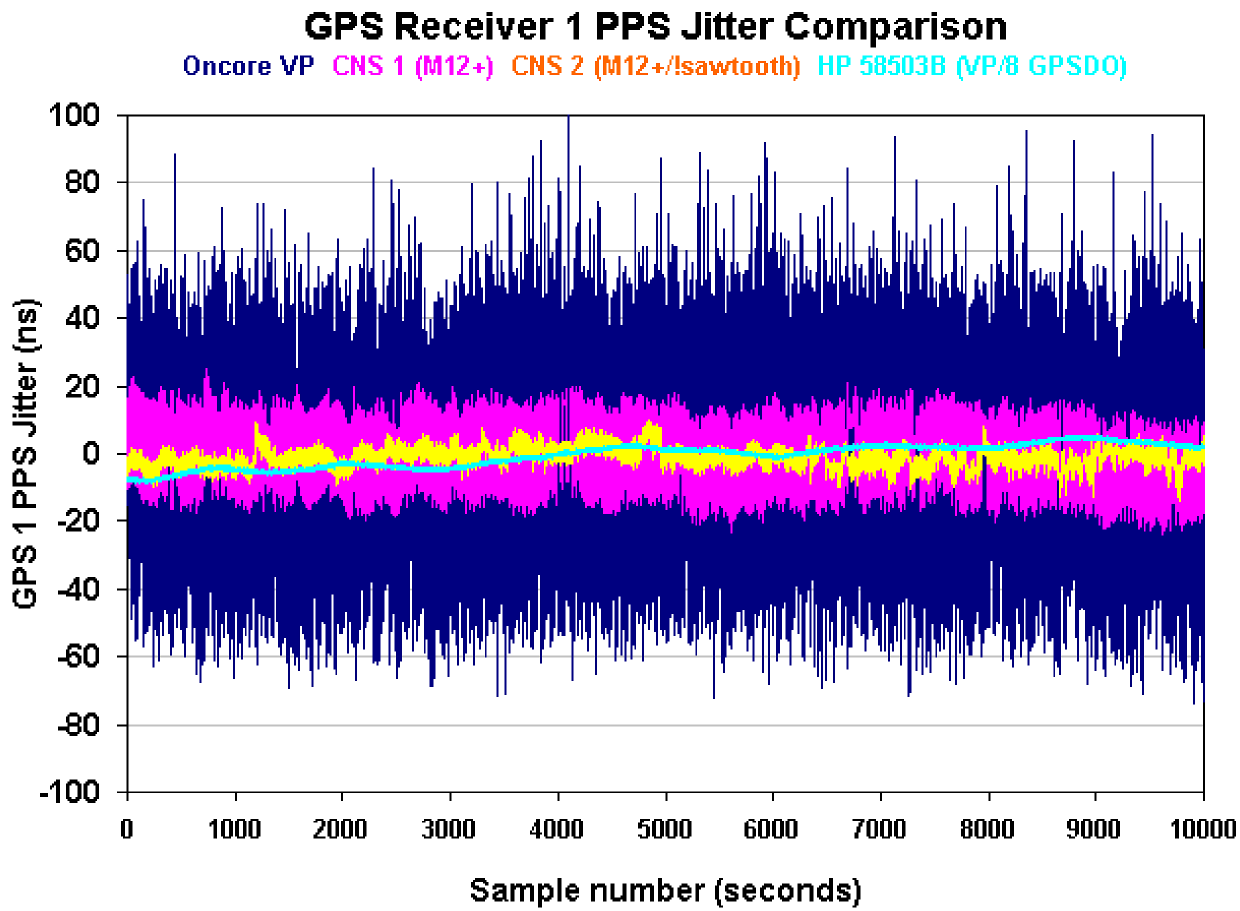
- Drive, of course
- This is America…

One-way or Round-trip

- Clocks run slower on a mountain so:
  - Measure frequency of clock at home
  - Measure frequency of clock on mountain
- Measure against what?
  - How about GPS time & frequency
  - Need better than 10^{-13} accuracy
  - Not with my GPS receivers

GPS reference

- Direct frequency measurement (GPS)



Summary – Big Idea

- 3 kids, 3 clocks, 1 family minivan
  - To 5400 feet
  - For 40 hours
  - Expect +20 ns
- Sync time/rate before trip
- Measure time/rate after trip
- Against reference; my "house standard"

Chapter 4

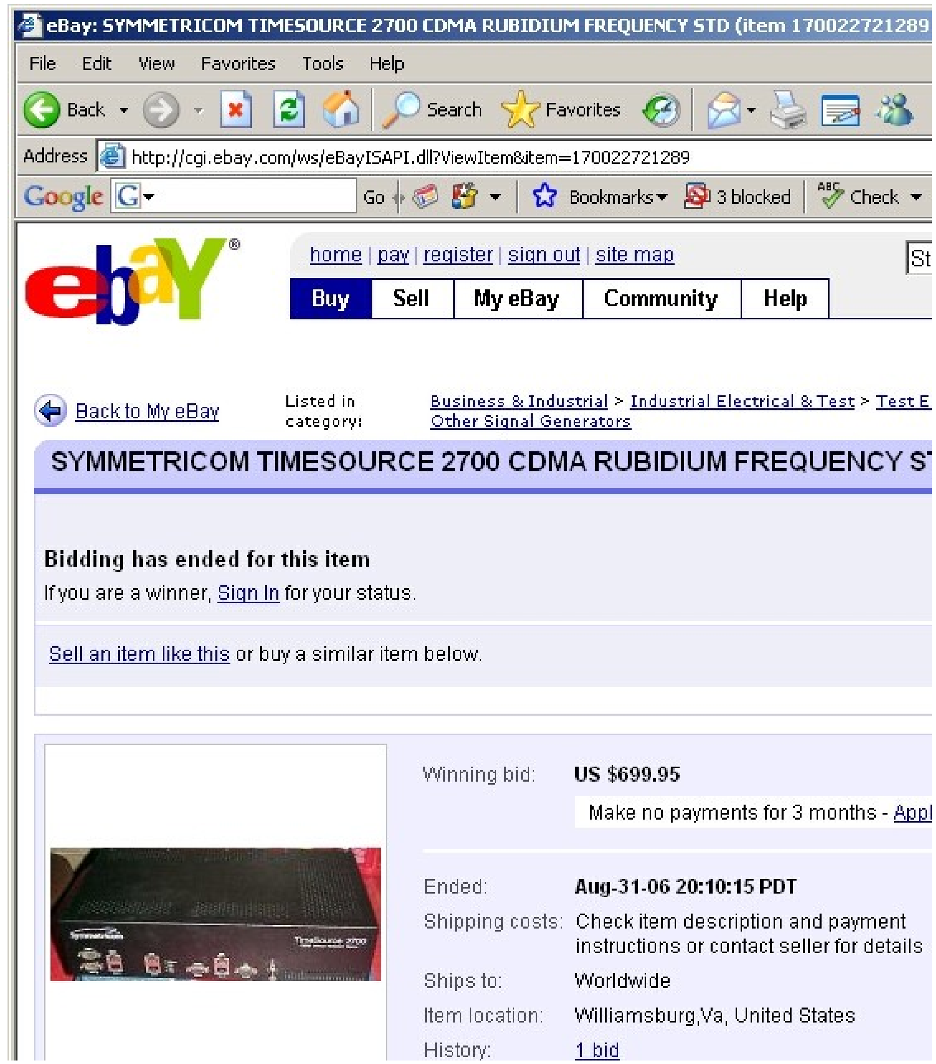
- Home time lab

Home Time Museum & Lab

Where does it all come from?

- Local aero/mil-surplus electronics
- Ham conventions; flea markets
- Used test equipment dealers
- Demo or refurbished models
- Friends, strangers; other "time-nuts"
- Sympathetic T & F companies
- And, of course, ebay® !

Surplus Time & Frequency



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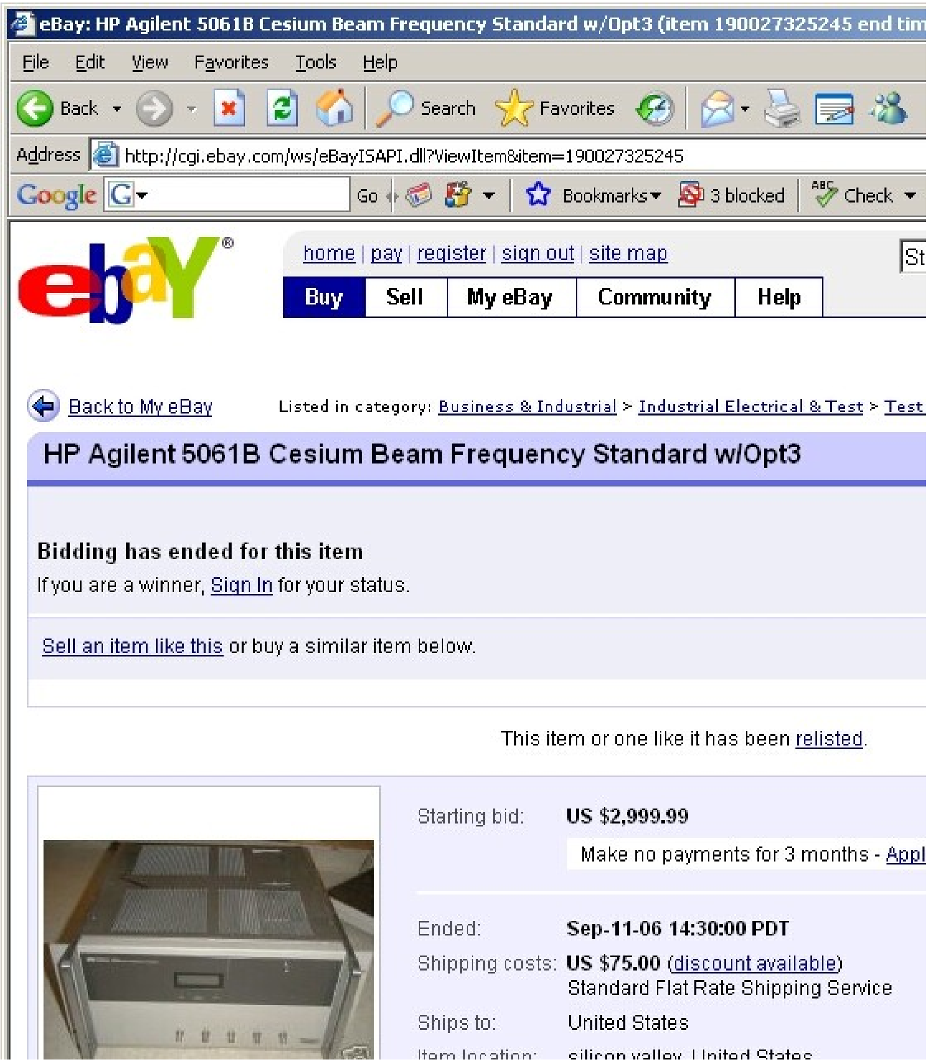

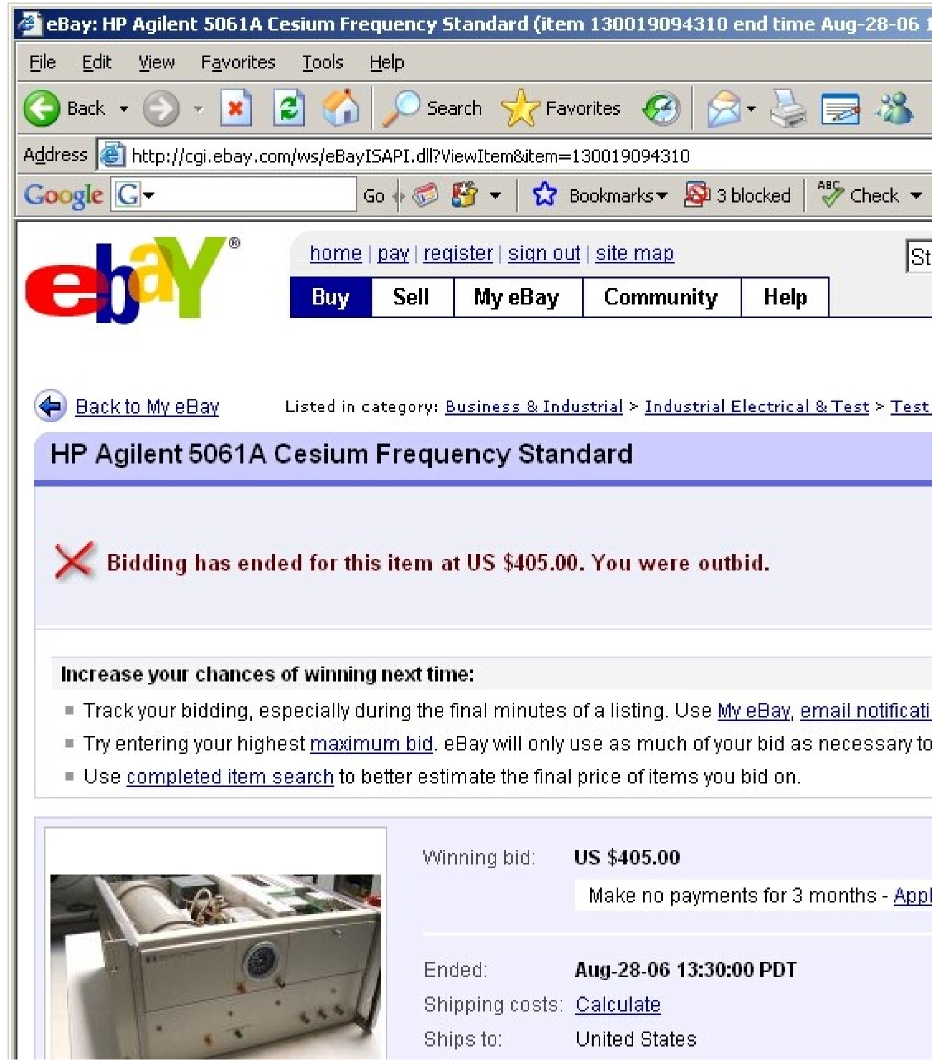

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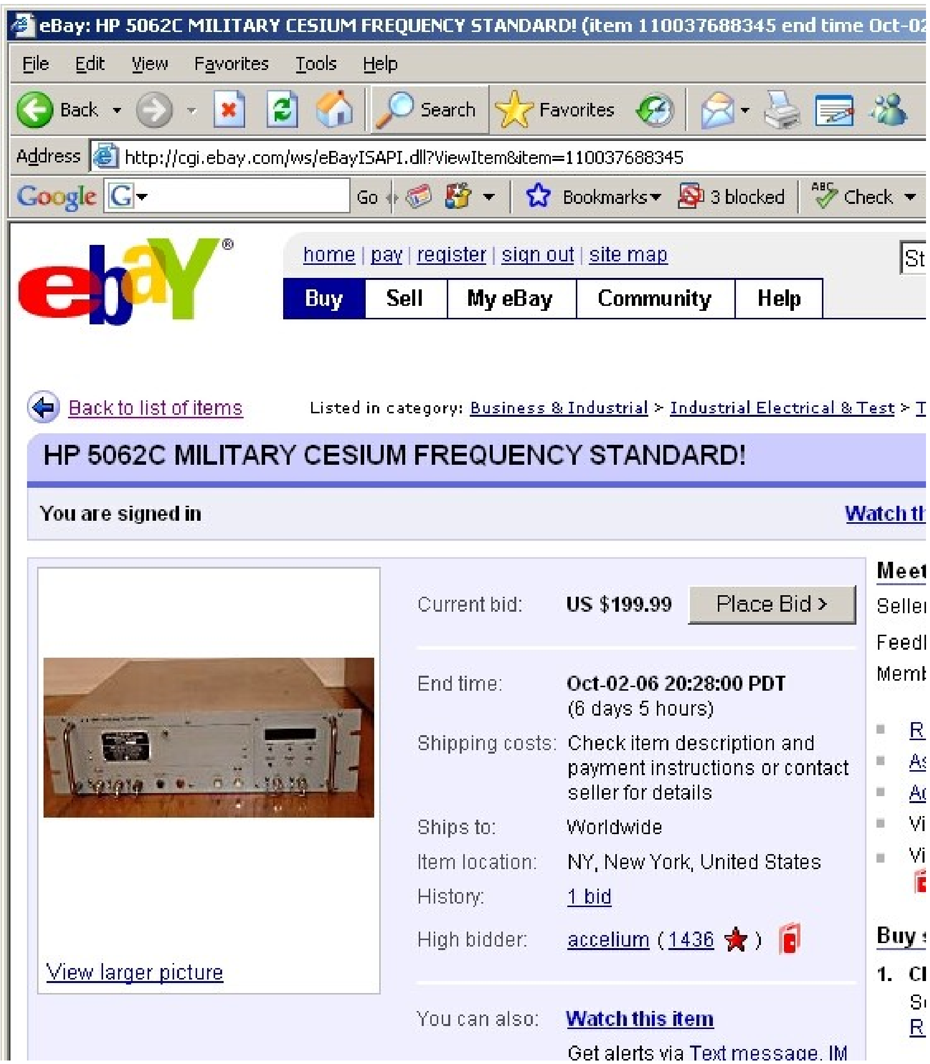

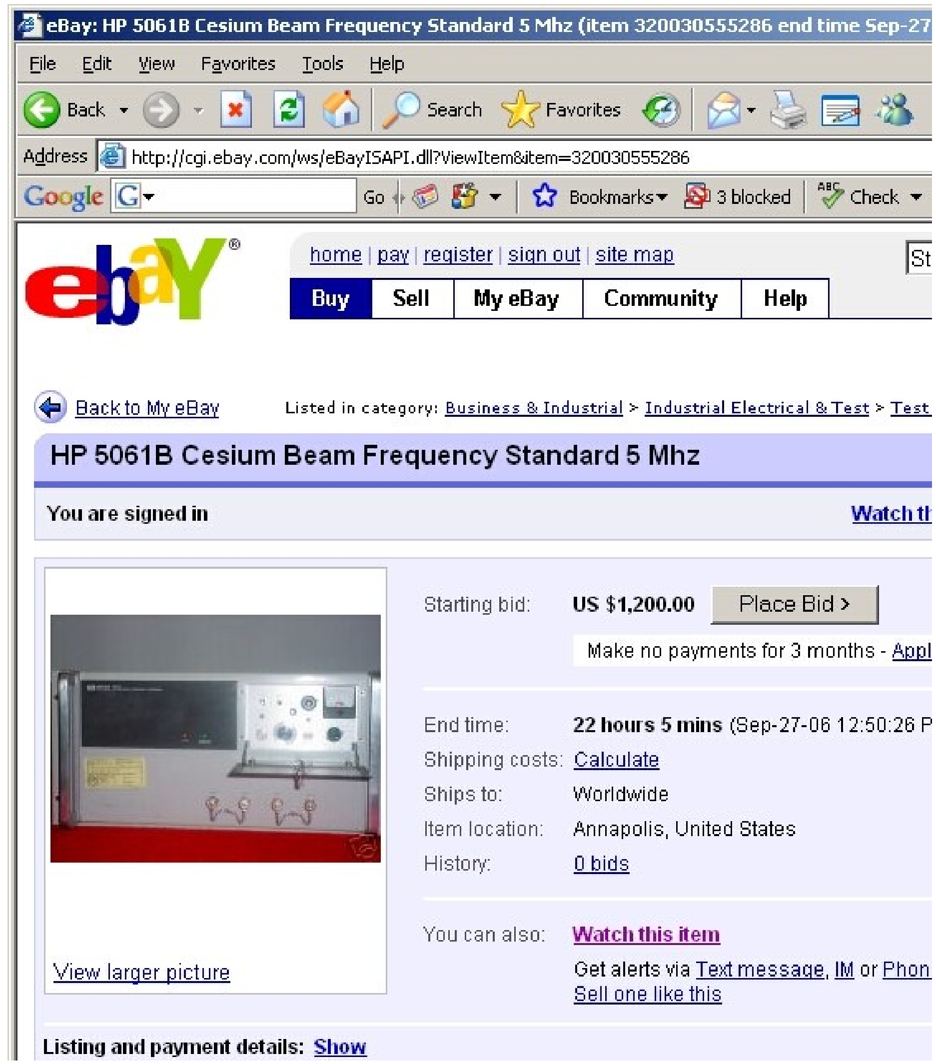

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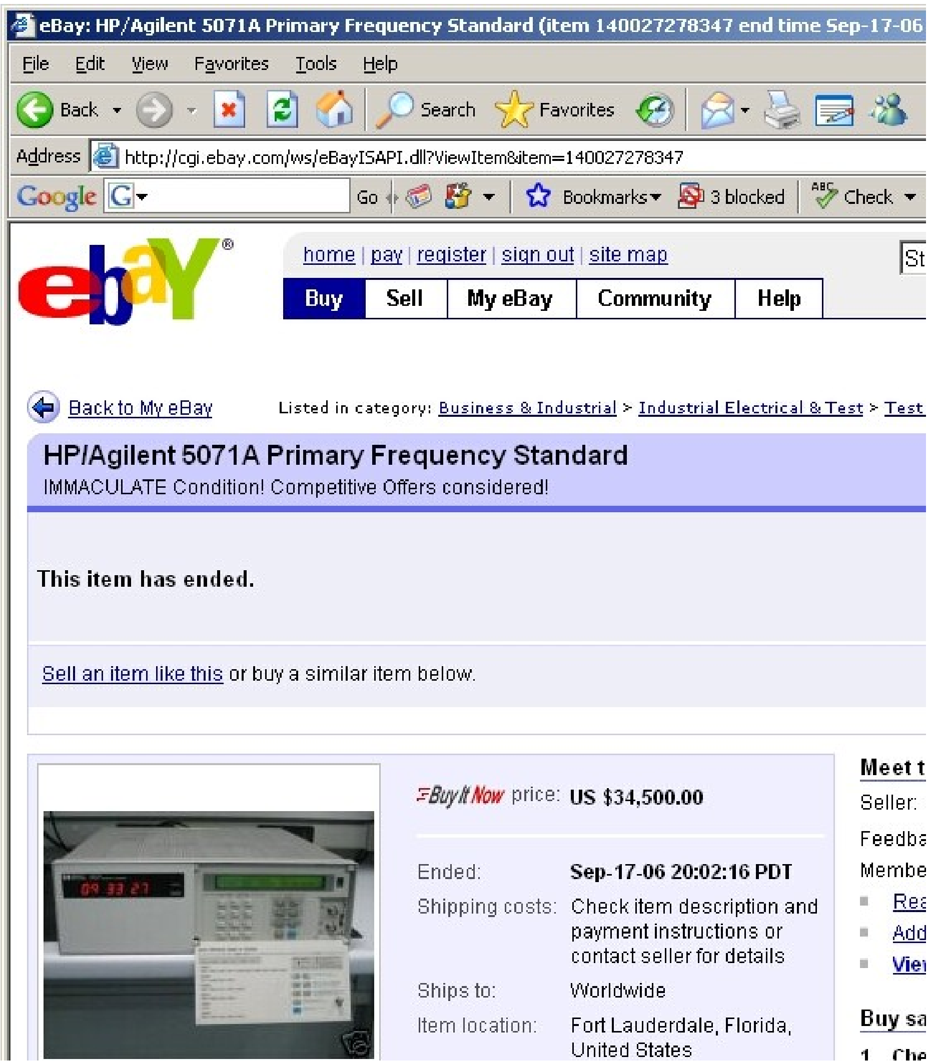

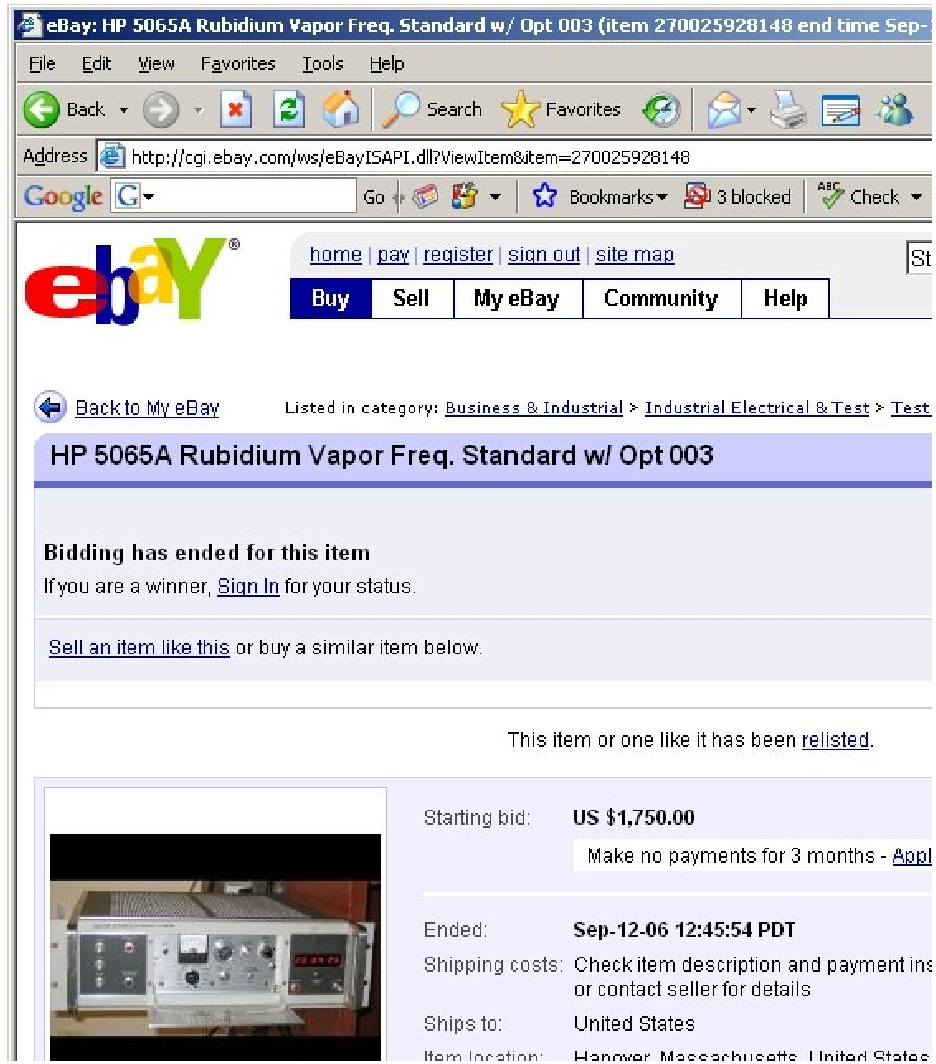

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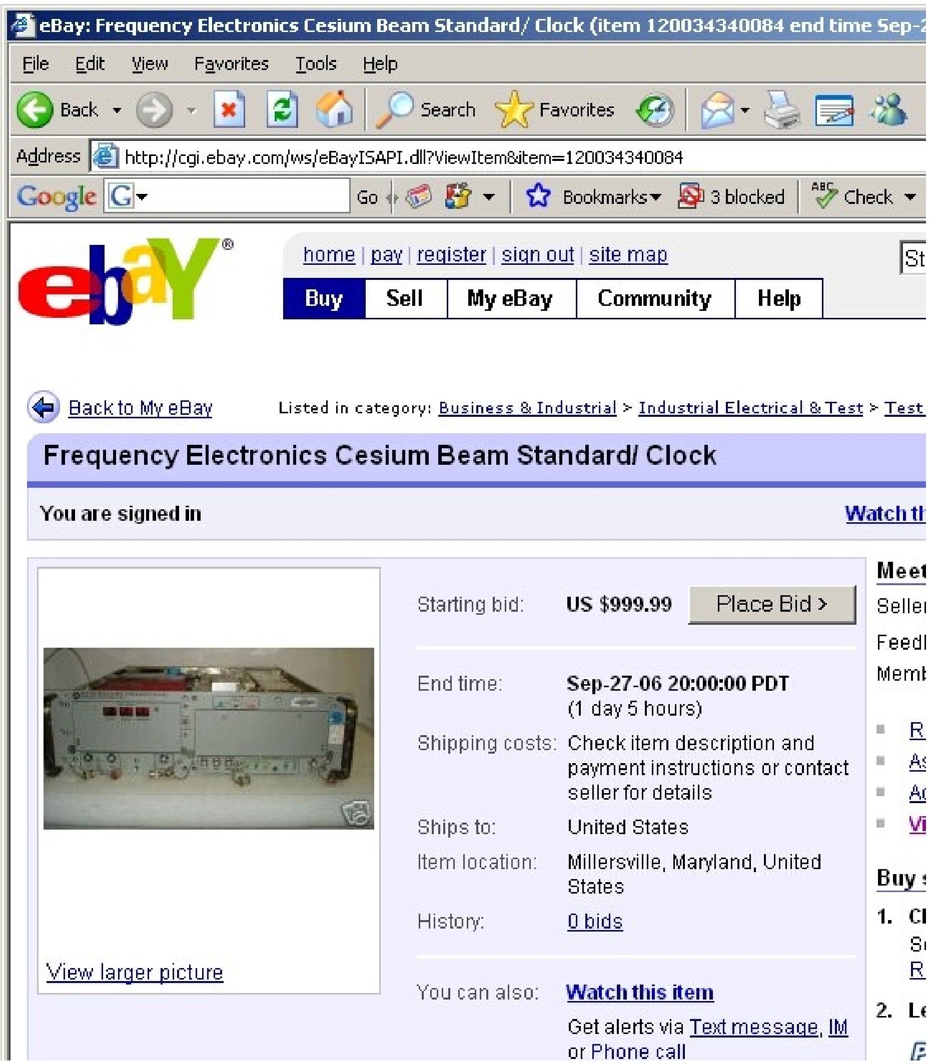

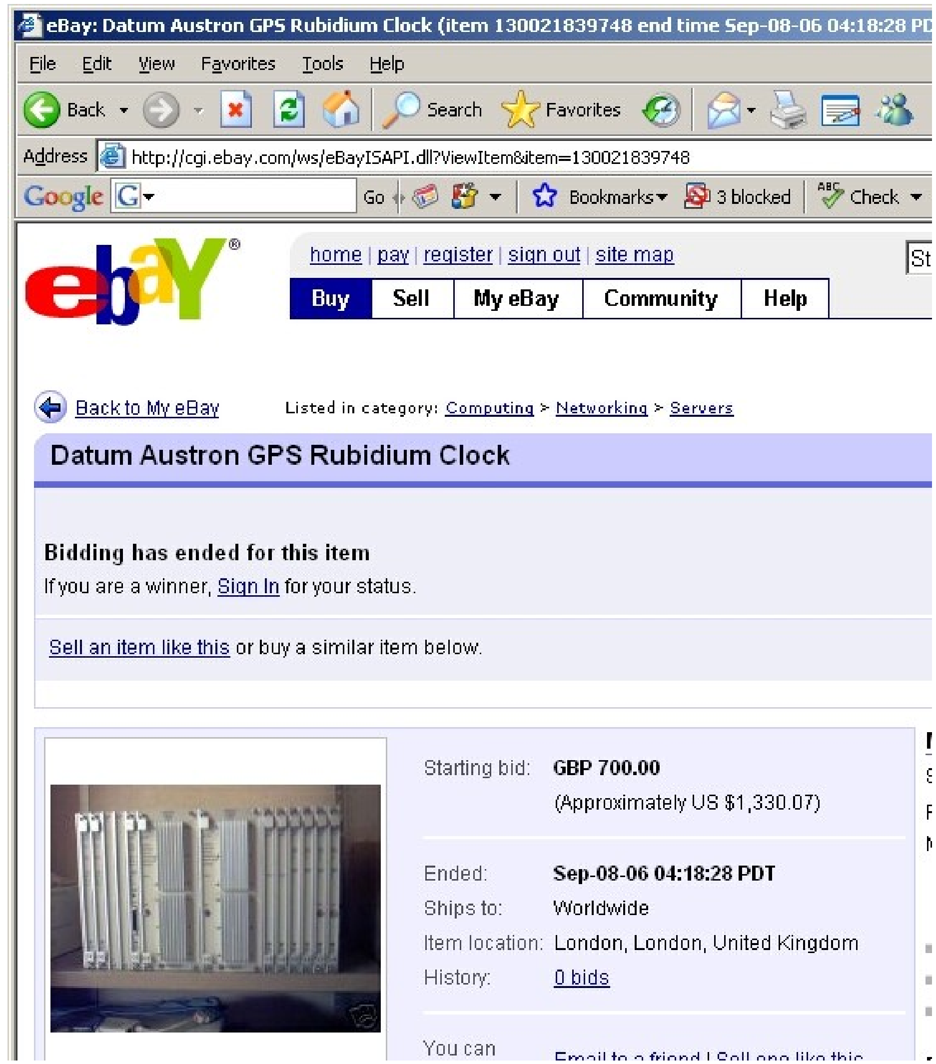

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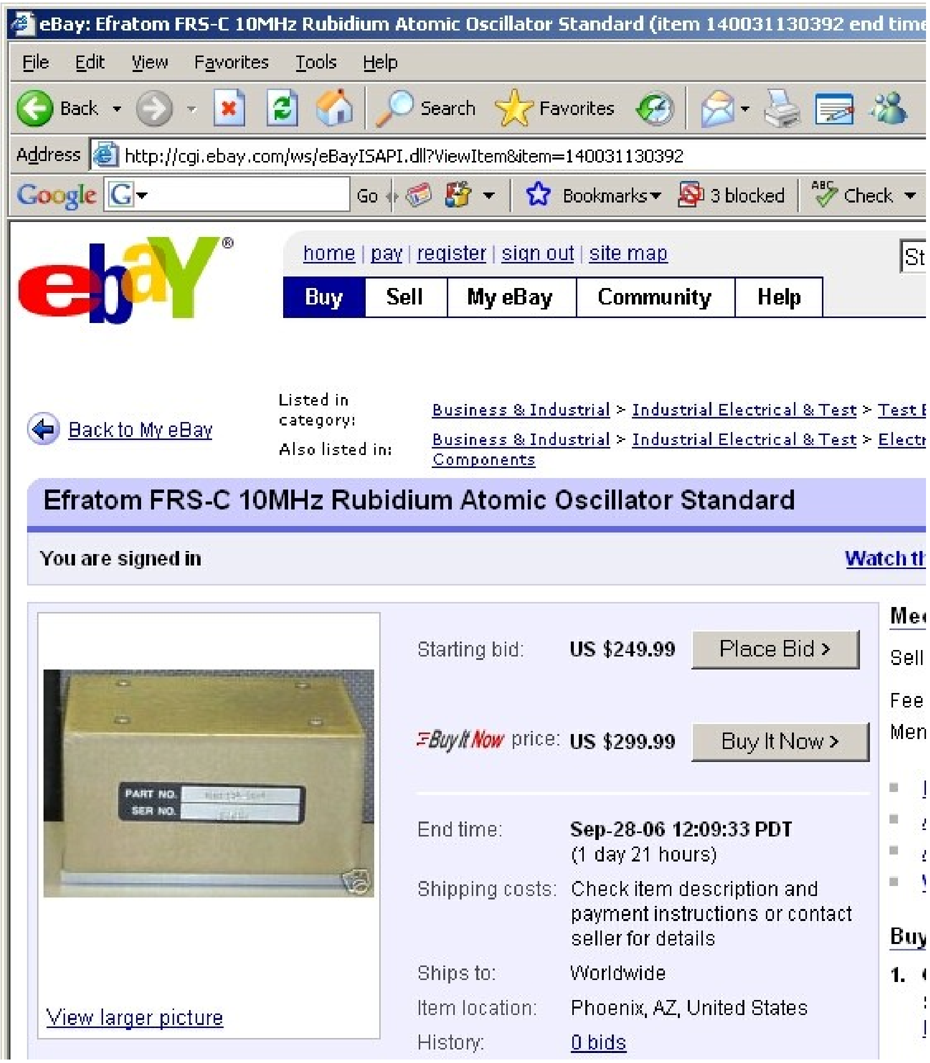

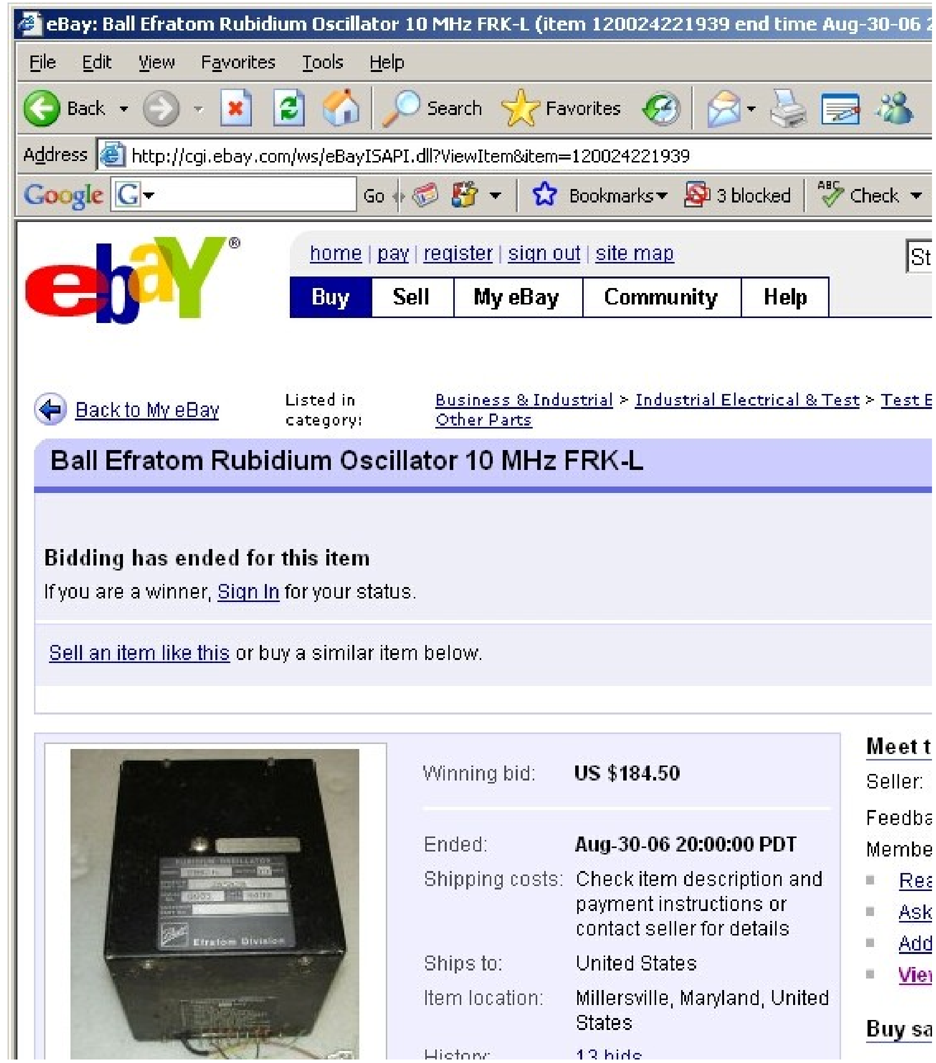

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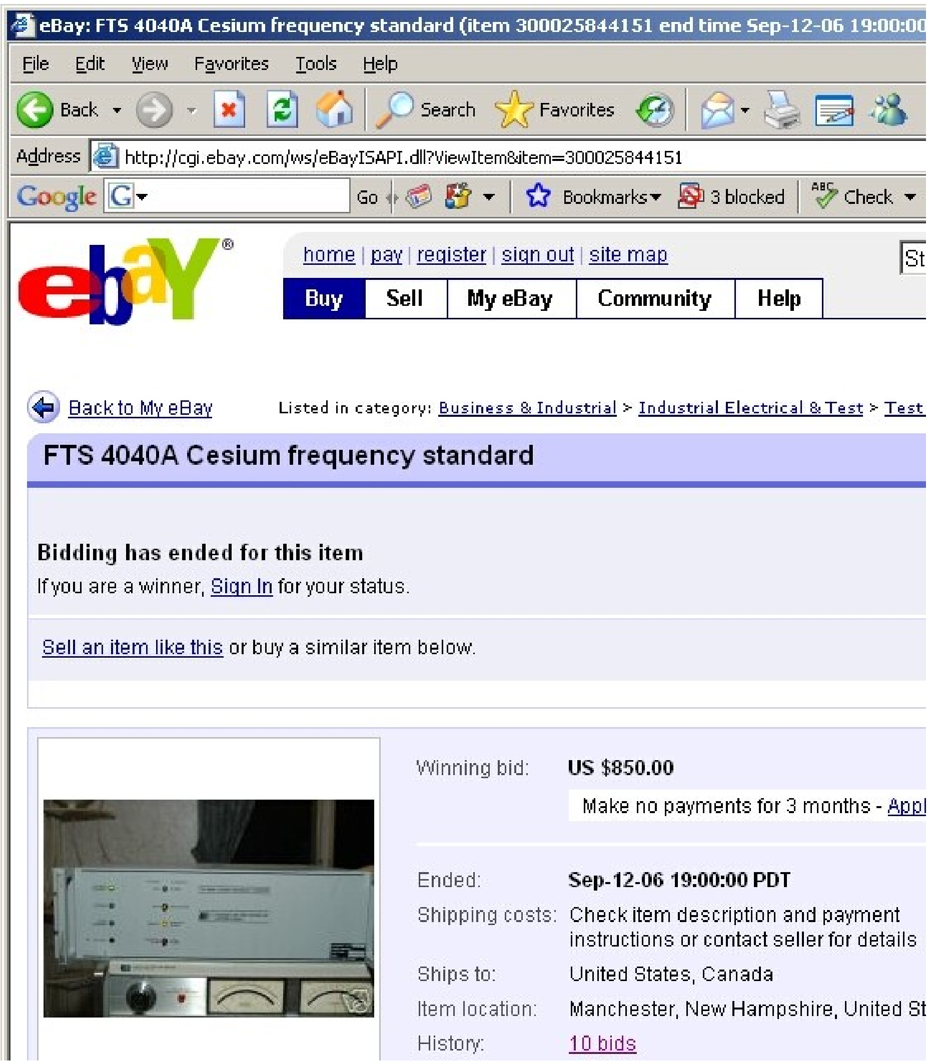

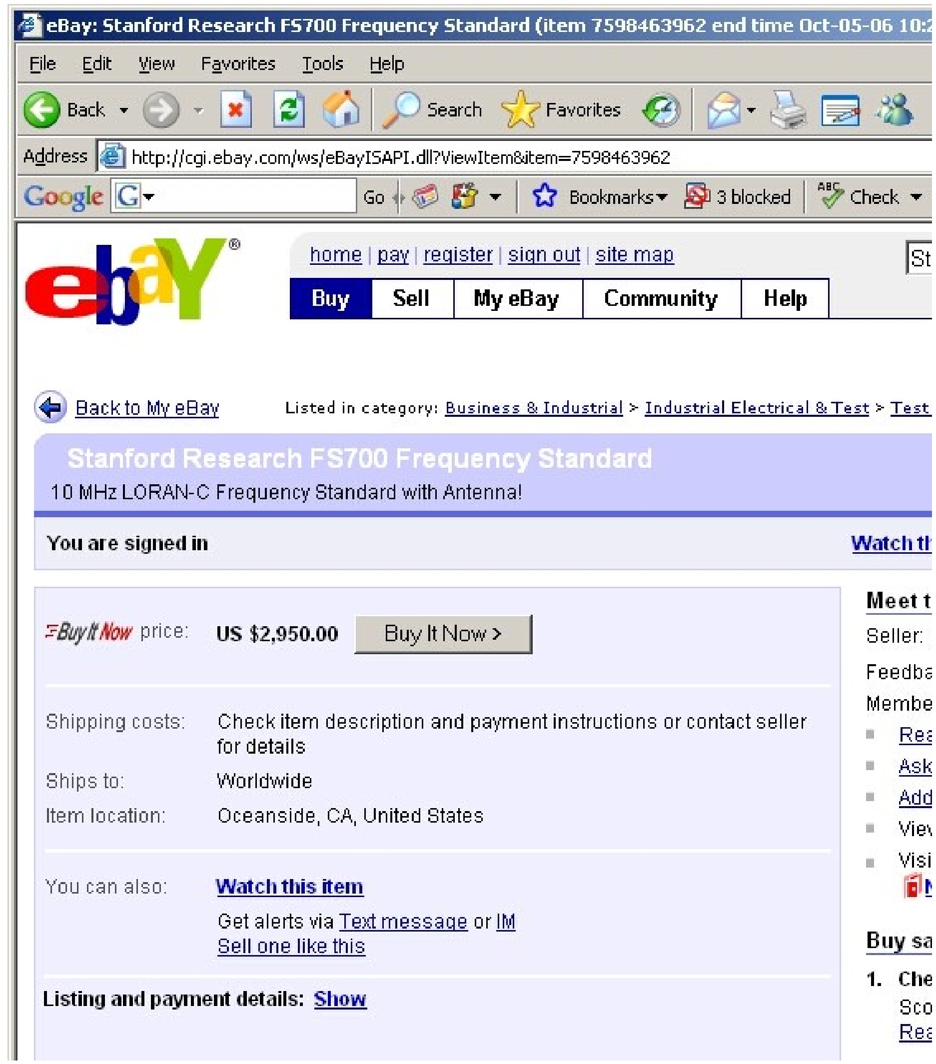

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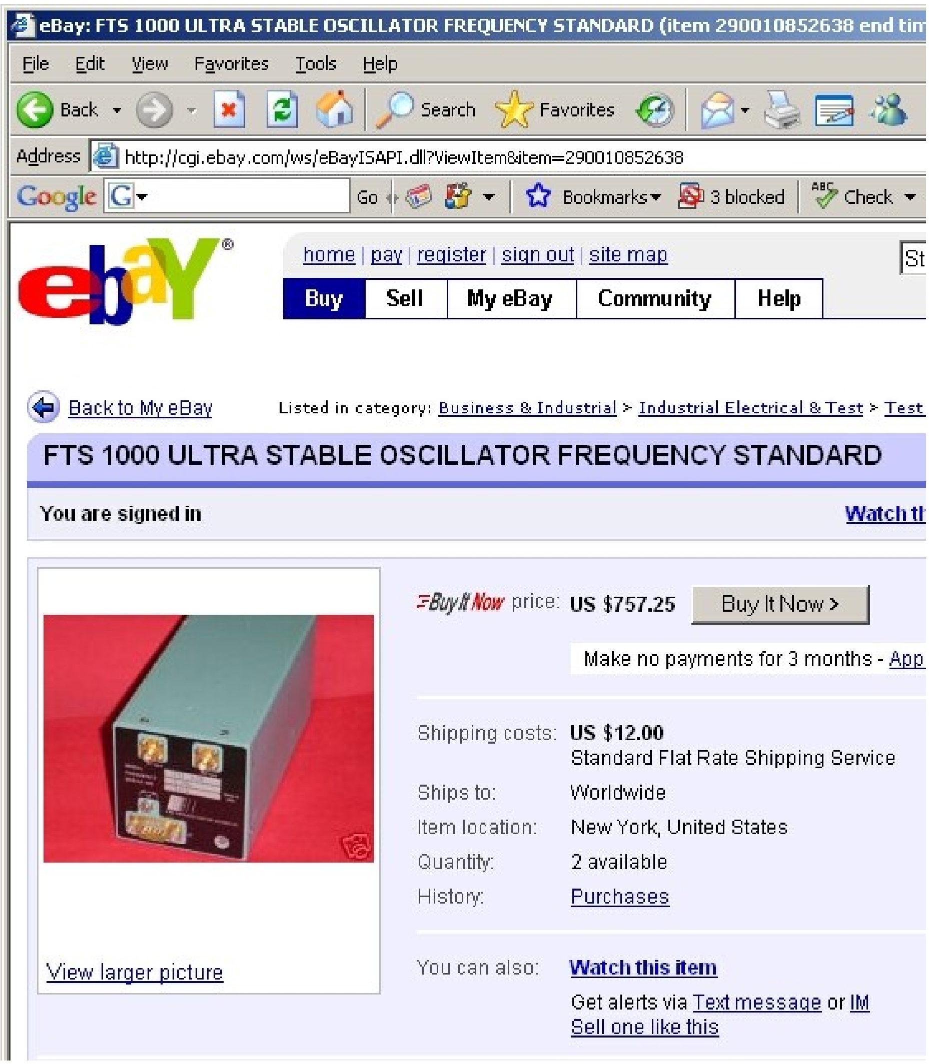

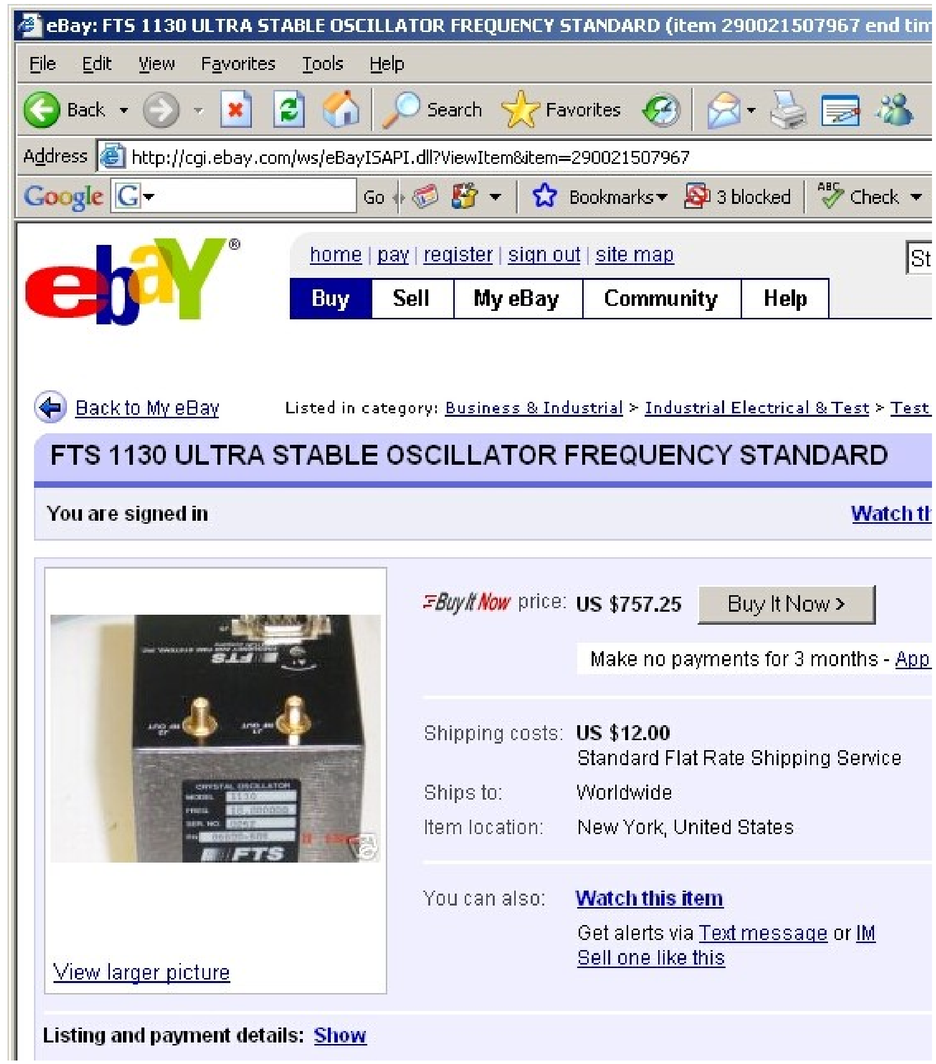

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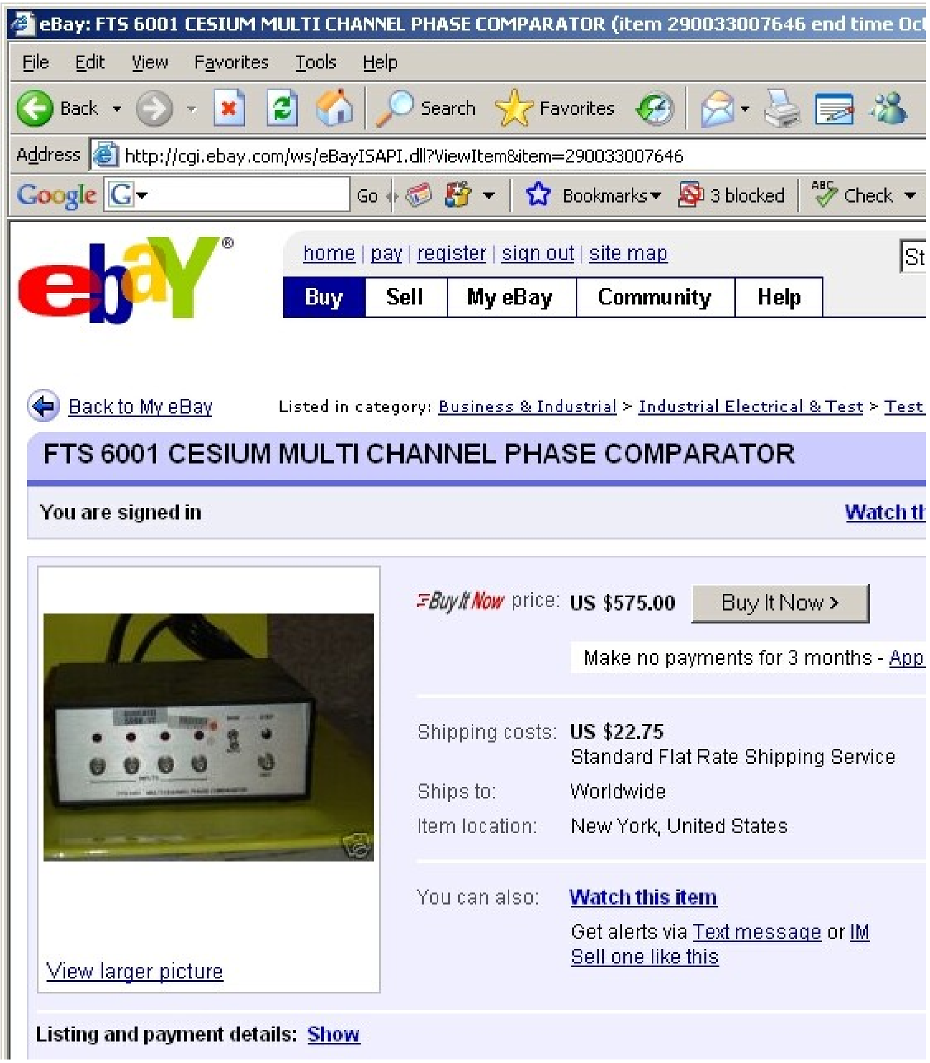

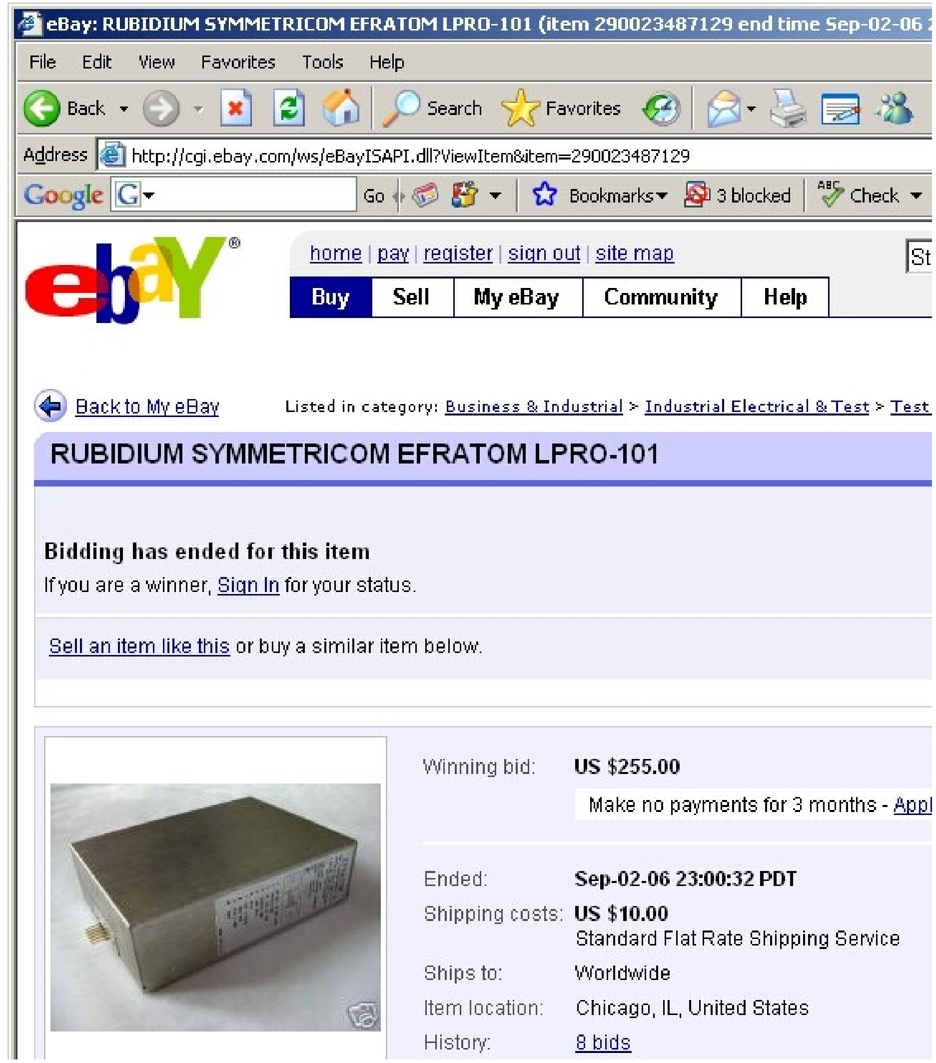

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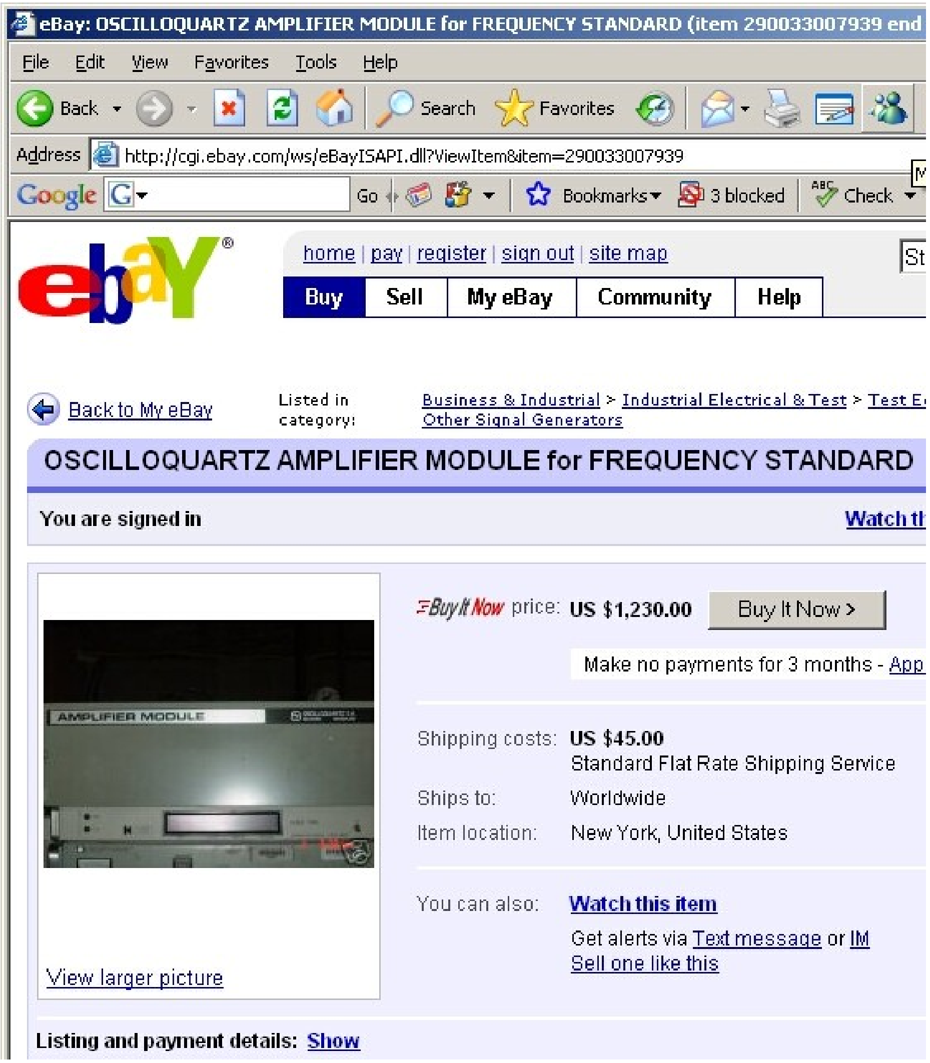

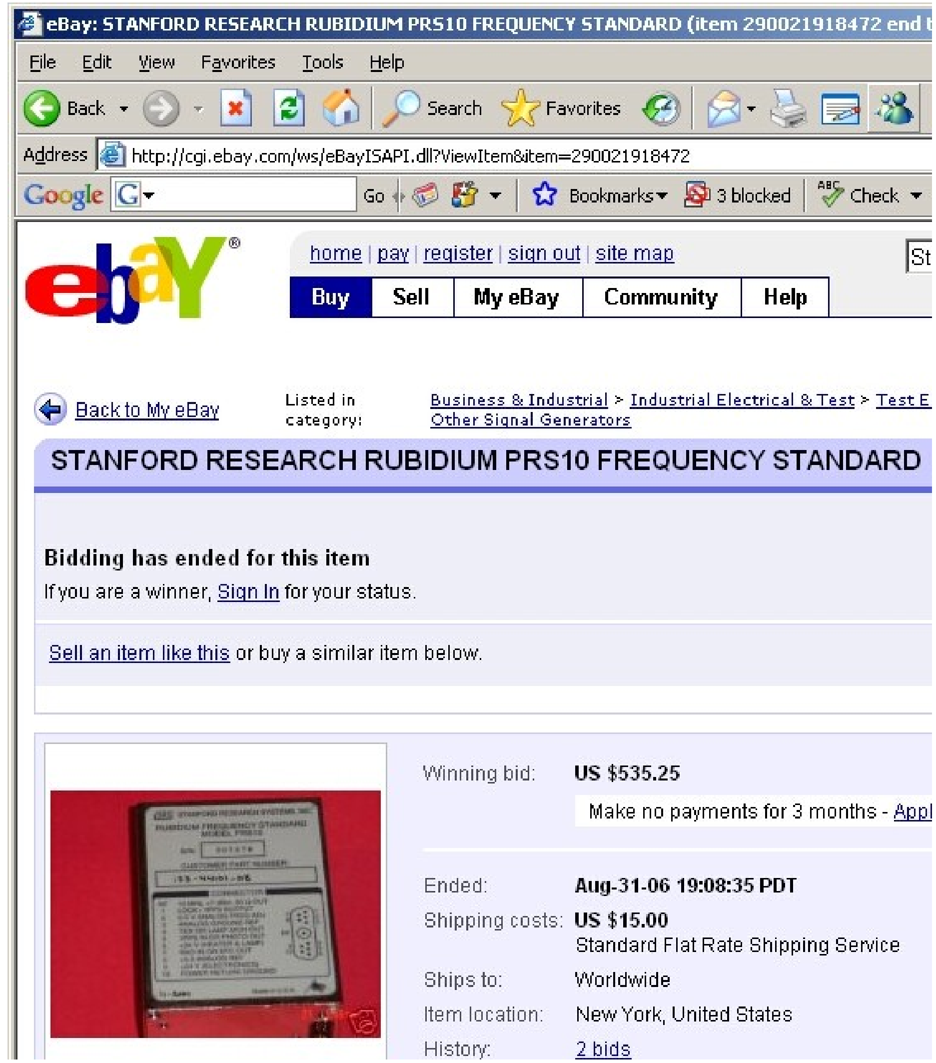

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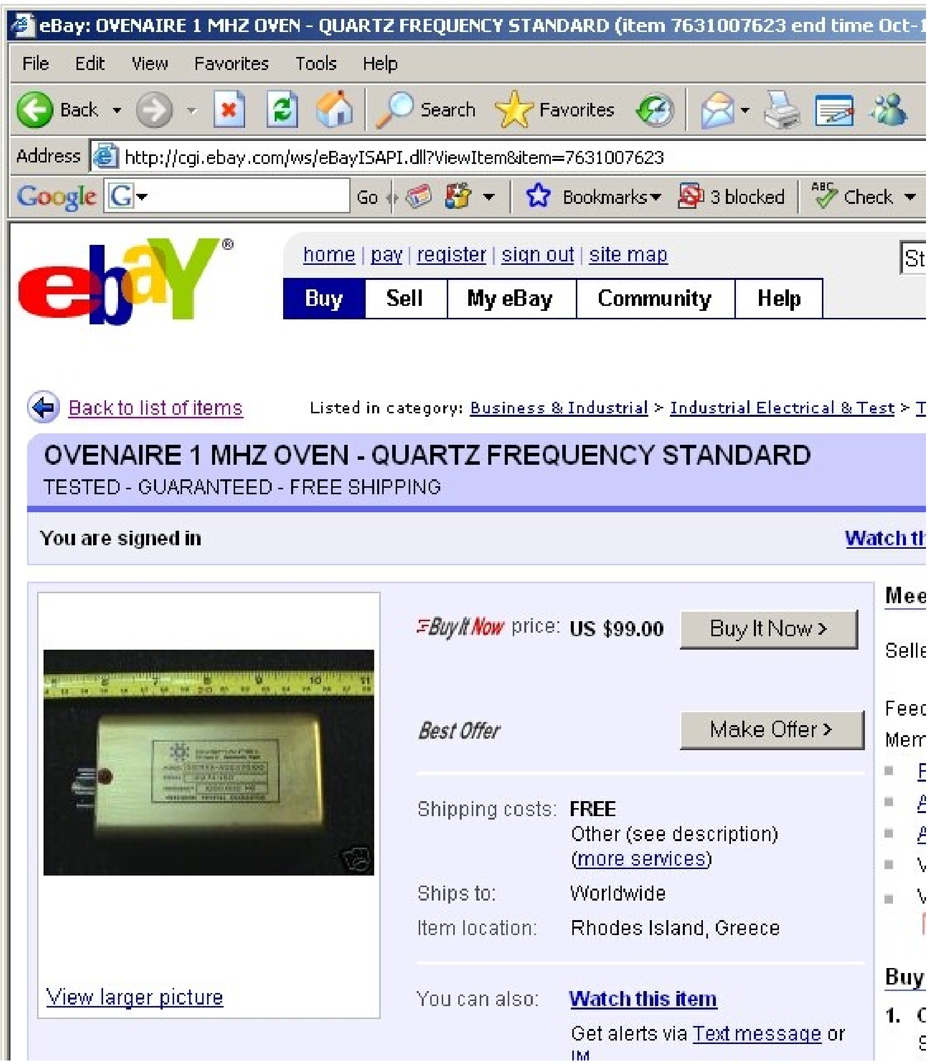

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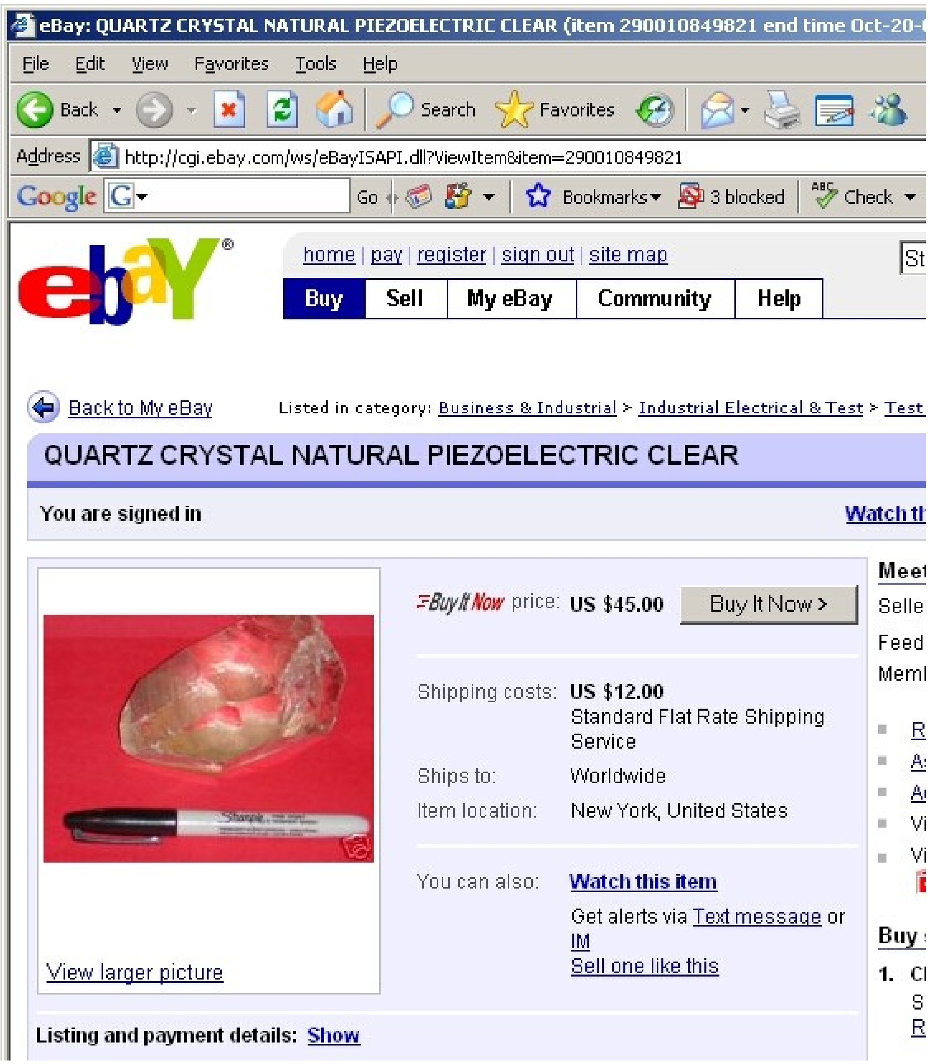

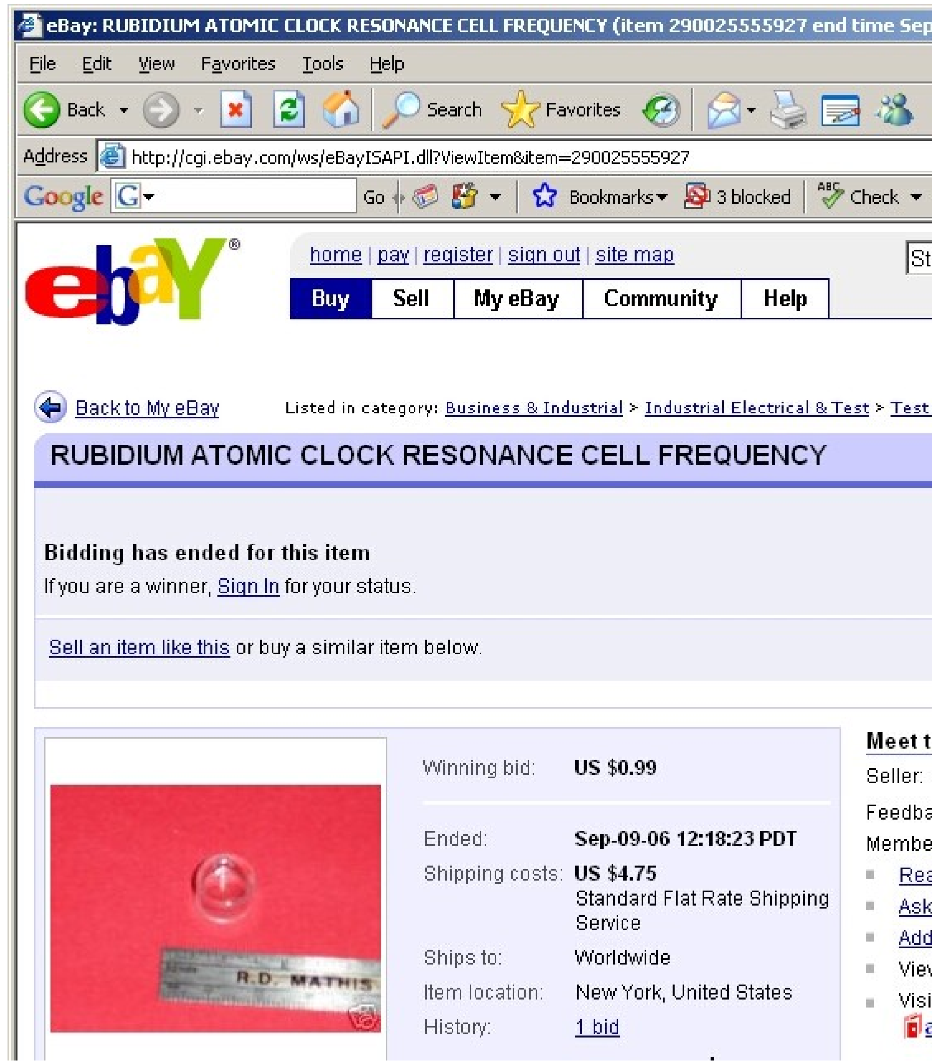

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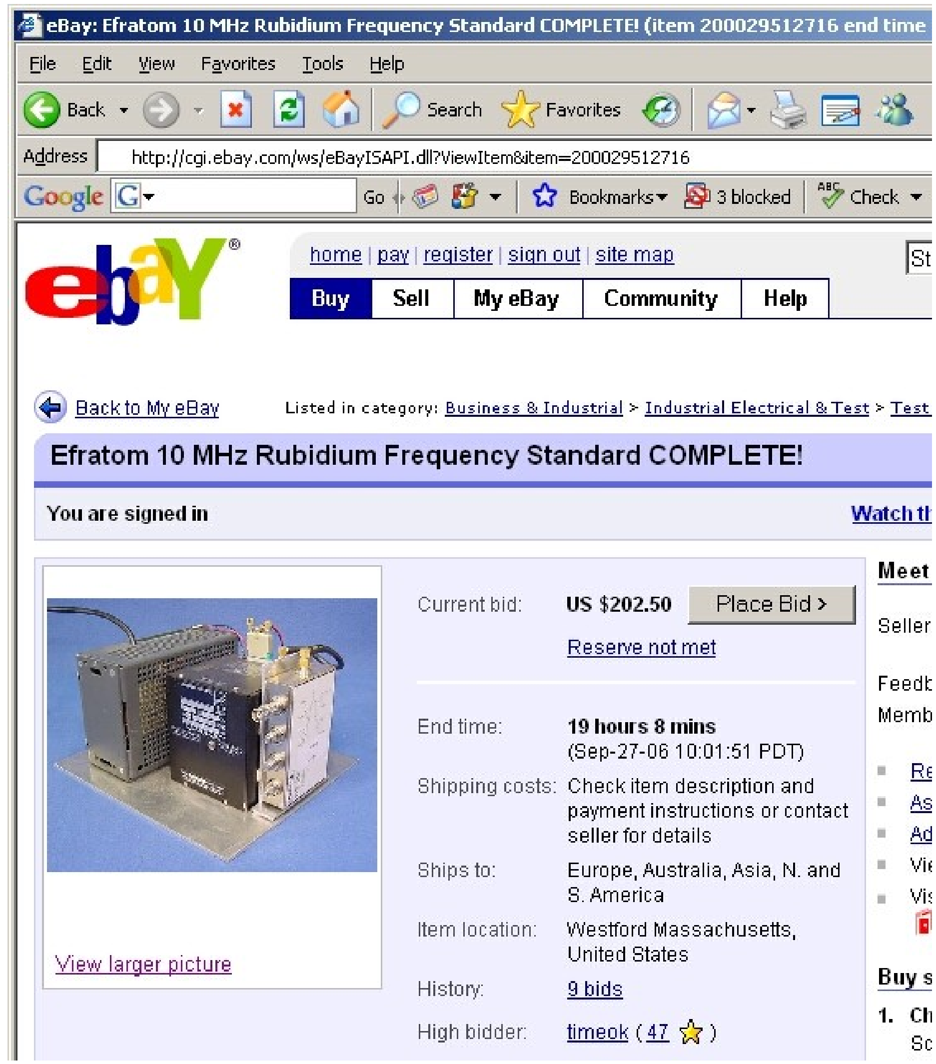

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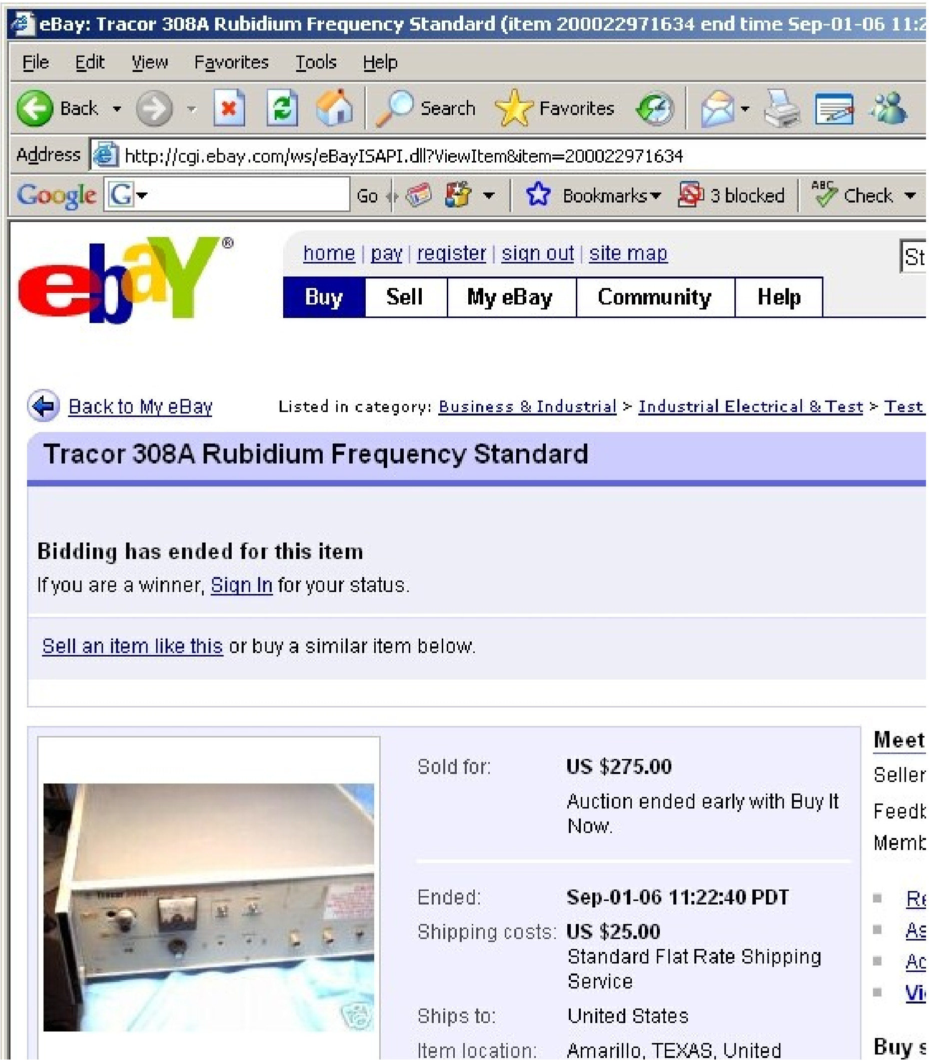
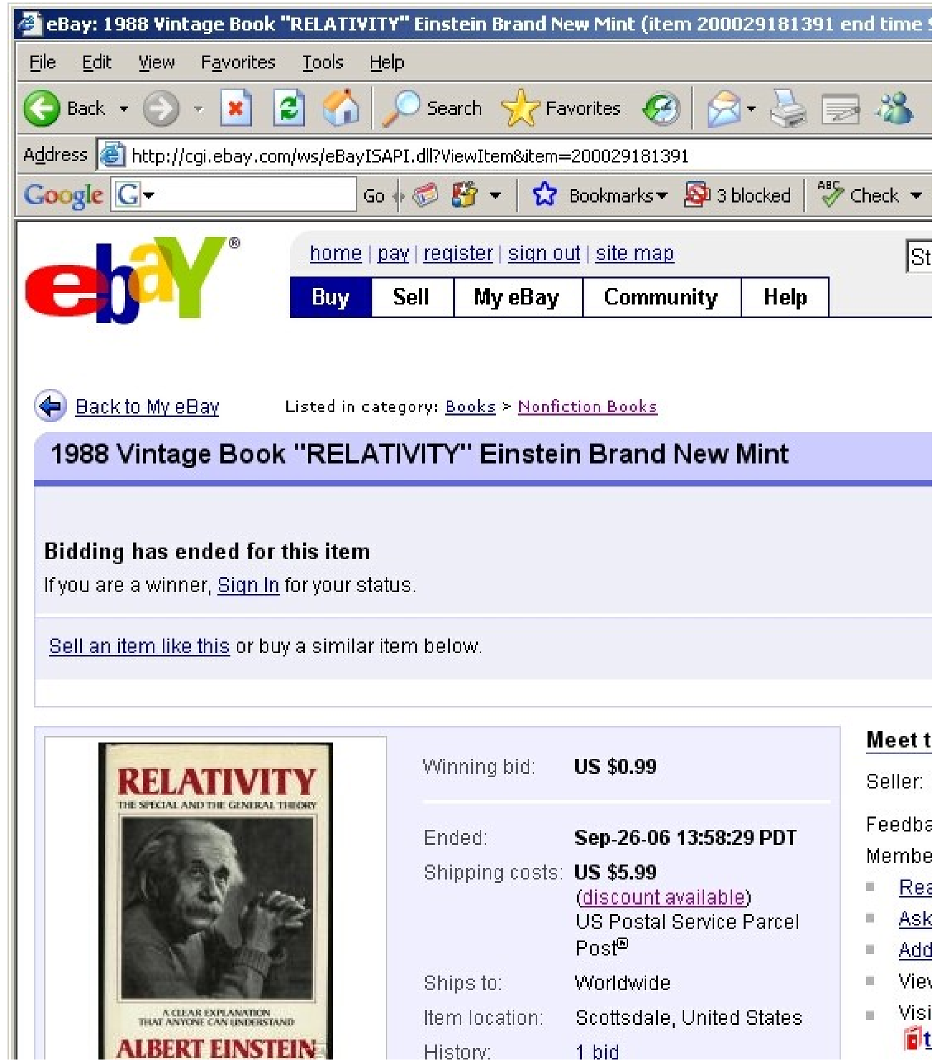




Home Time Lab

- Not just dirt cheap clocks, but…
- Manuals, cables, connectors, power supplies, meters, GPIB, USB, software, GPS antennas, GPSDO, WWVB, Loran-C, clocks, displays, IRIG, TCG, frequency counters, TIC, phase comparators, etc.
- Most old, some new; possibly broken, often working. Buy 3, play Frankenstein.

Museum of HP Clocks

Requirements

- GR effect 1.1×10^{-16} / m
- $\Delta f = 1.5\times10^{-13}$
- If we want 10% accuracy
- We need a reference good to 10^{-14}

Chapter 5

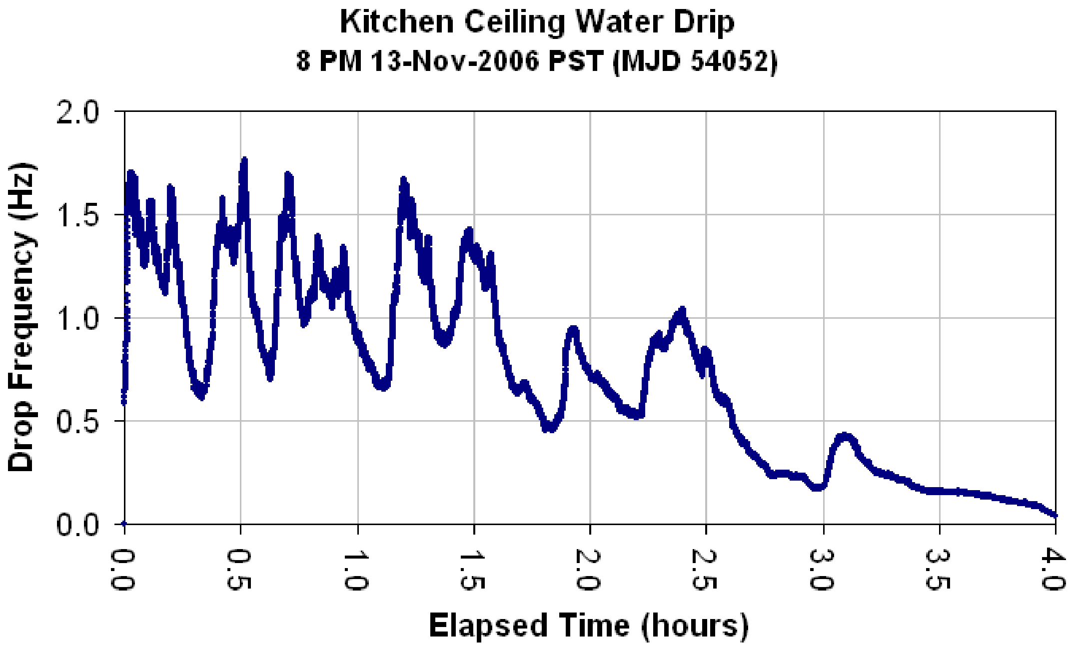
- A Powers of Ten tour

Powers of Ten - Introduction

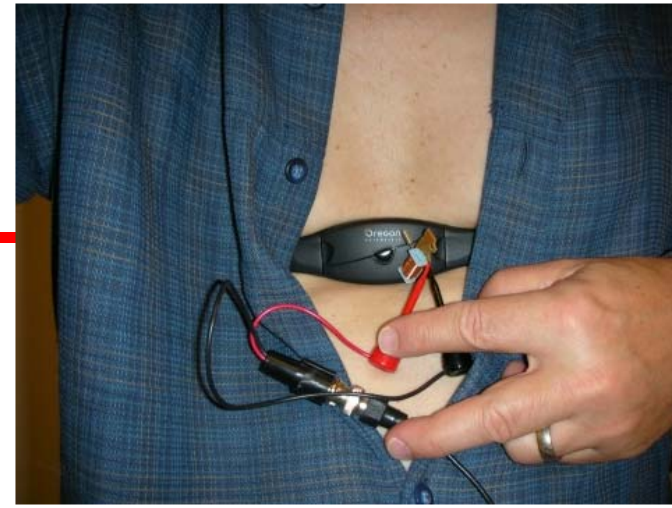
- A quick overview of clock accuracy
- What clocks keep poor time?
- What clocks keep best time?
- And many in between…

10^{-0} drip, drip

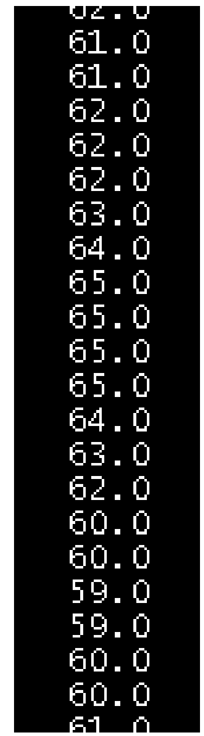
- Leak in ceiling
- 0.57 s … 9.9 s
- 1.7 Hz … 0.1 Hz



10^{-1} heart beat

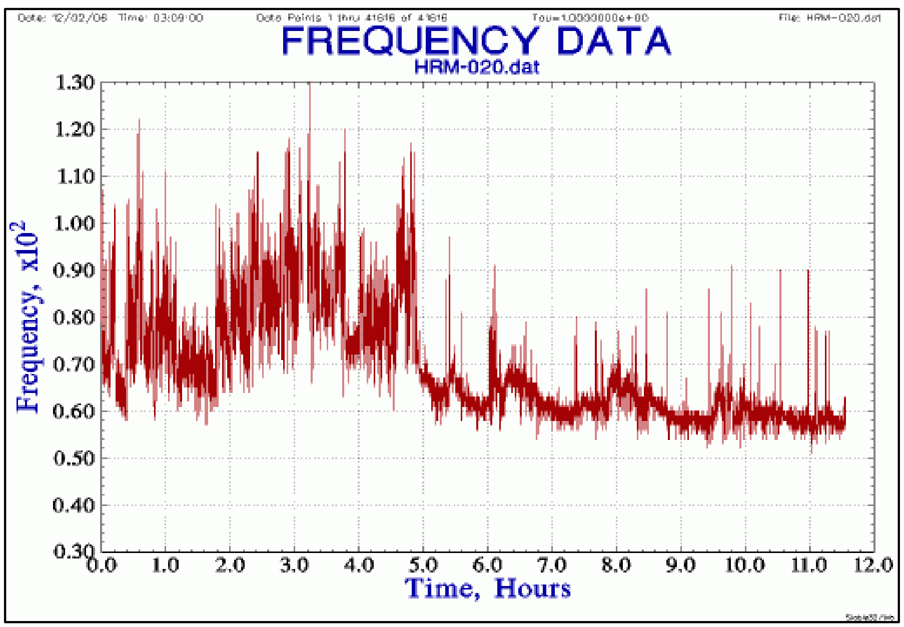


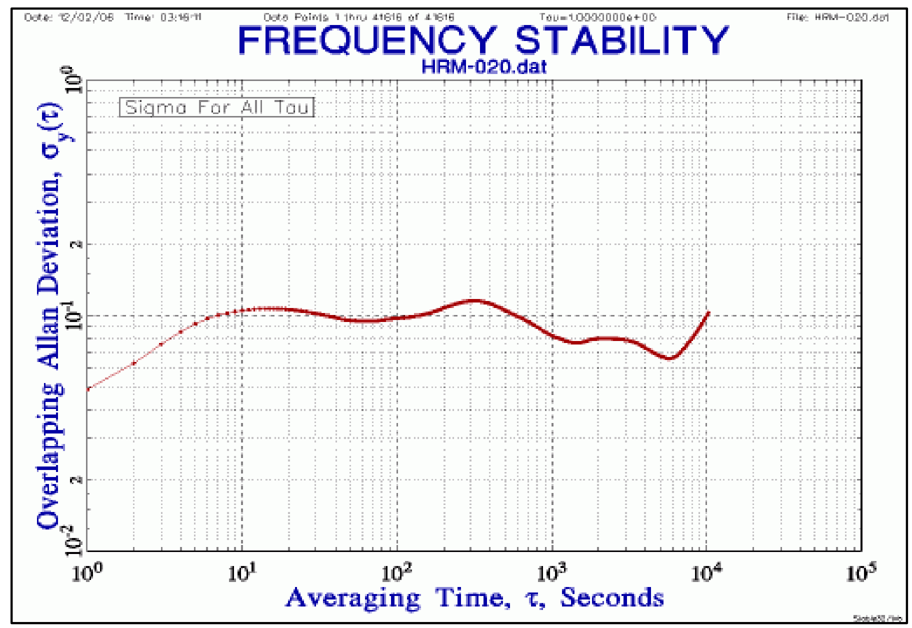
- 10^{-1}, 0.1, 10%
- The original '1 PPS'
- Sometimes 2x, even 3x
- Much higher stability at night
- < 10% accuracy possible



10^{-1} heart beat

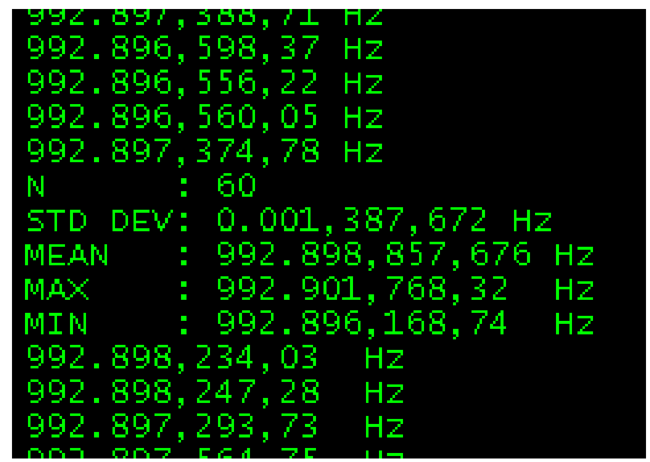
- 12 h frequency plot (evening/night)
- ADEV floor is 10^{-1} from 10^1 to 10^4 s!
- (is this OK?)



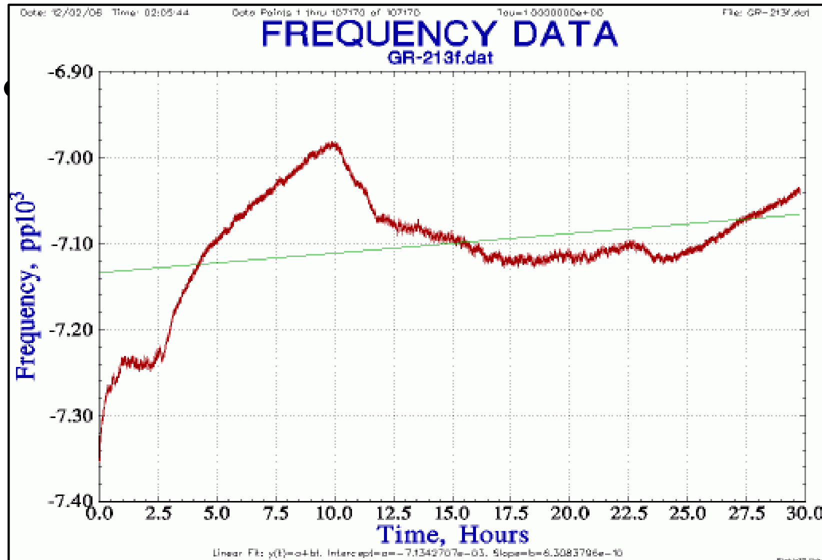


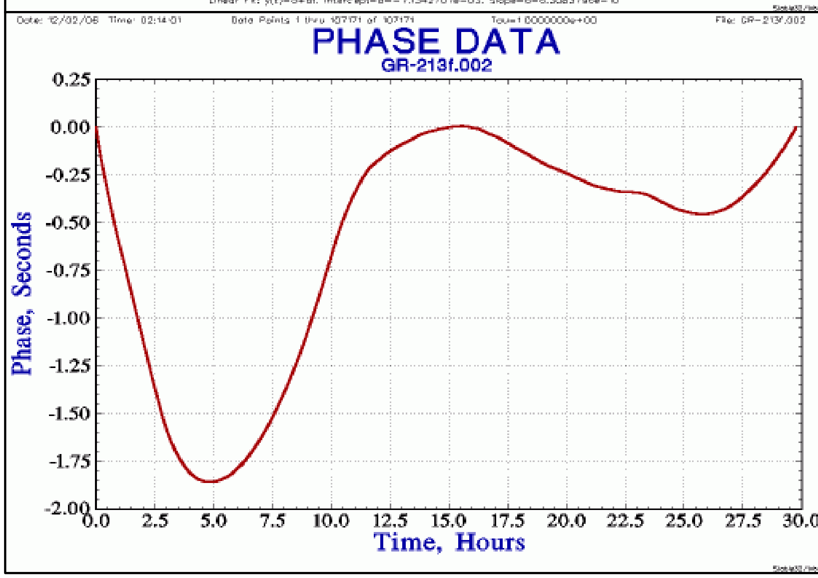
10^{-2} tuning fork oscillator

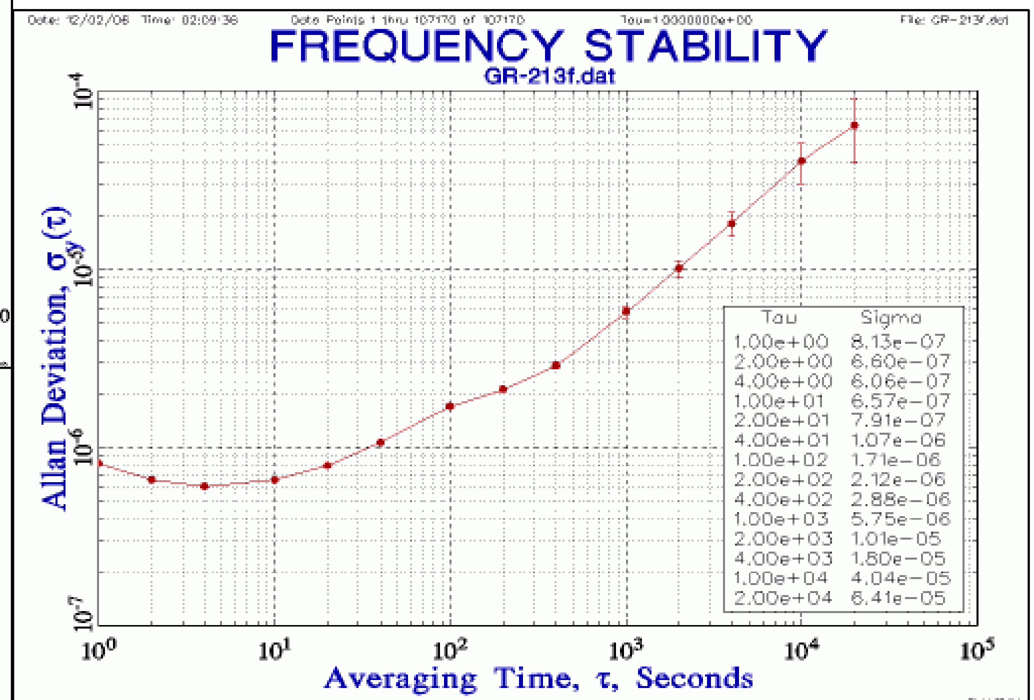
- 0.01, 1%
- General Radio Type 213 Audio Oscillator
- 1 'kc'; f = ~992.8 Hz
- ±1.3 mHz (60 x 1 s)
- Accuracy < 1%
- Count those 9's
- ADEV is 10^{-6}



10^{-2} tuning fork oscillator

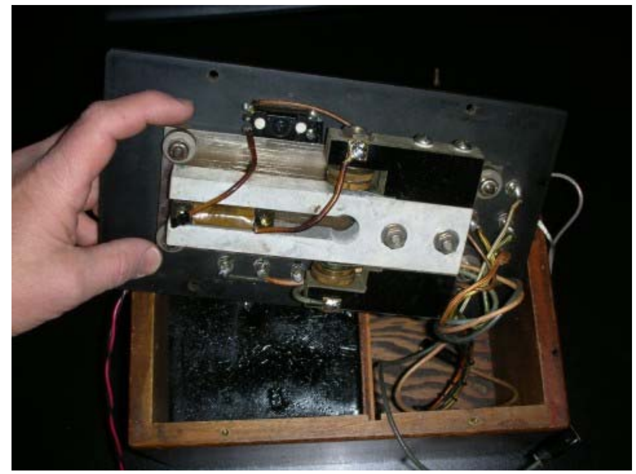


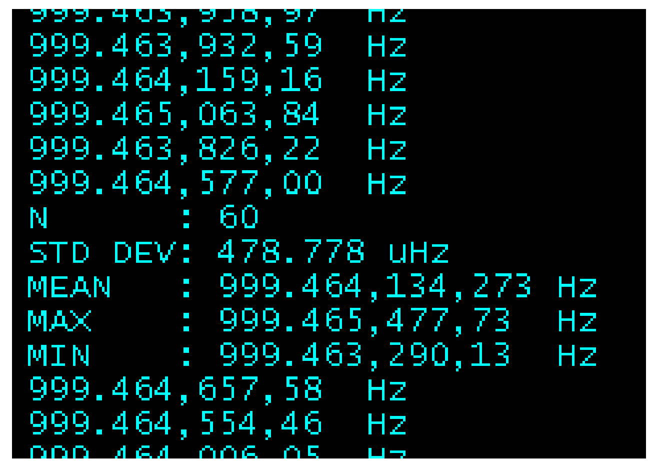




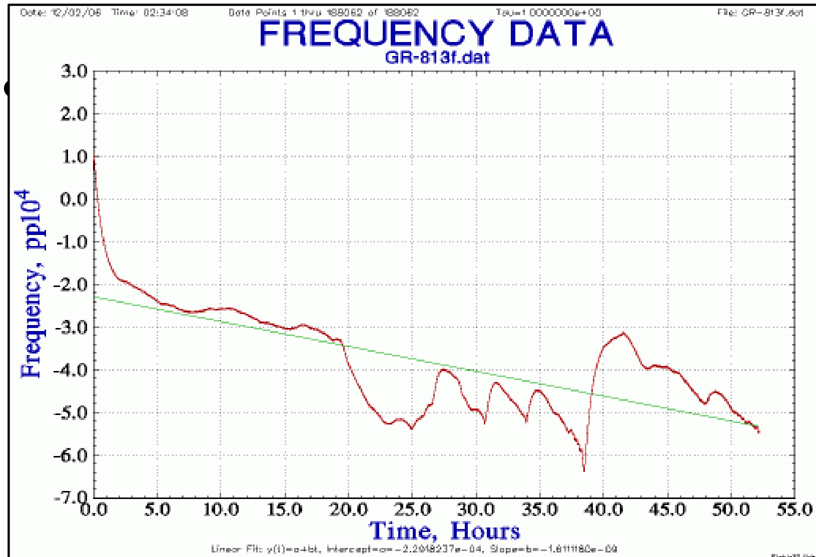
10^{-3} precision tuning fork

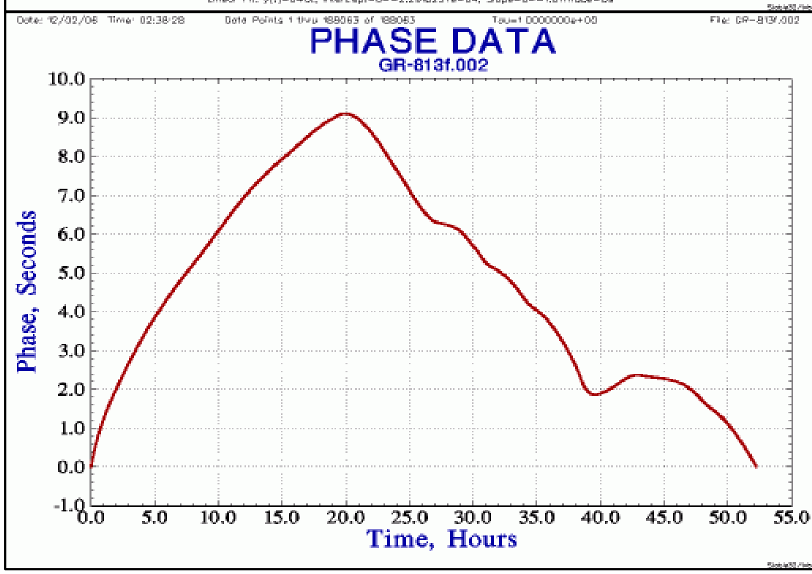
- 0.001, 0.1%, 1 ms/s
- General Radio Type 813
- 1 'kc' tuning fork
- f = ~999.4 Hz
- ±400 μHz (60 x 1 s)
- Accuracy < 0.1%
- ADEV is 10^{-7}

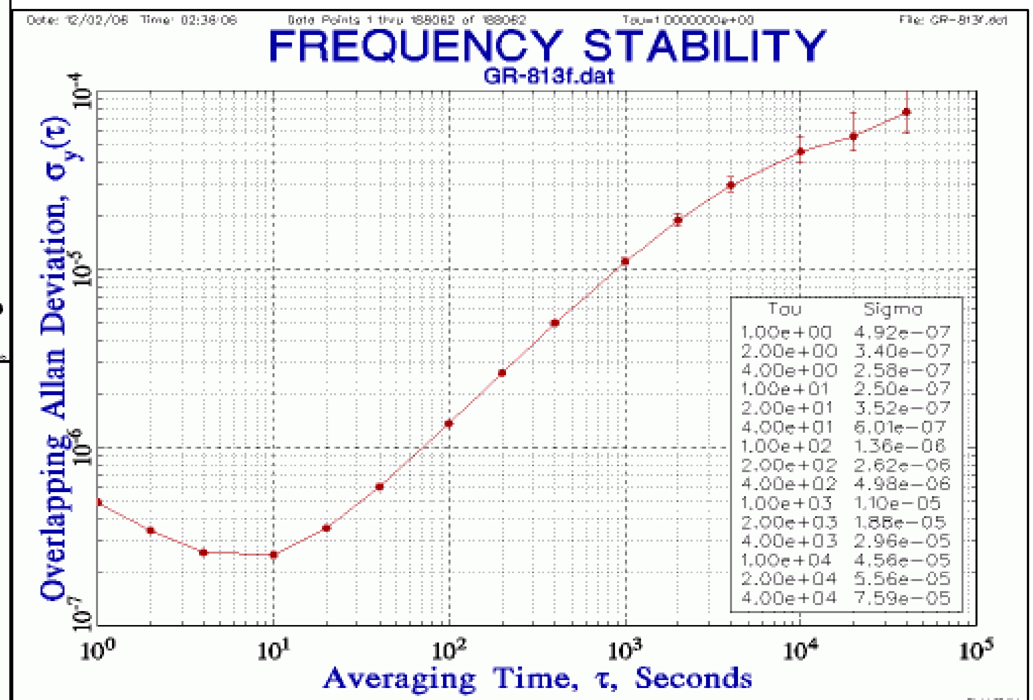




10^{-3} precision tuning fork

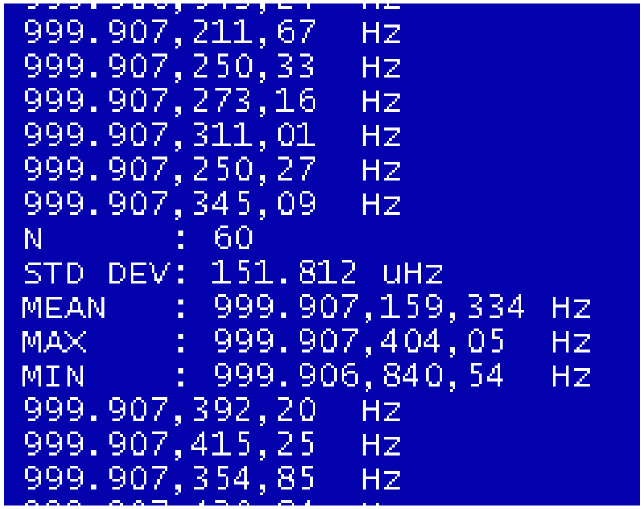


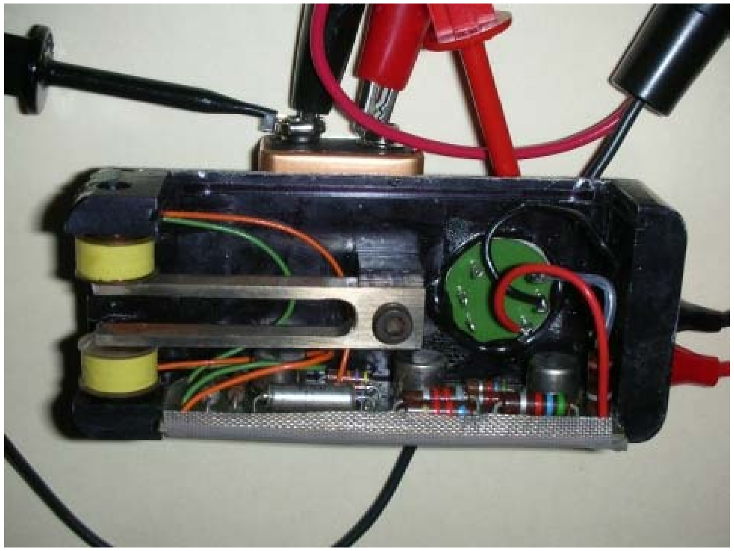




10^{-4} mechanical oscillator

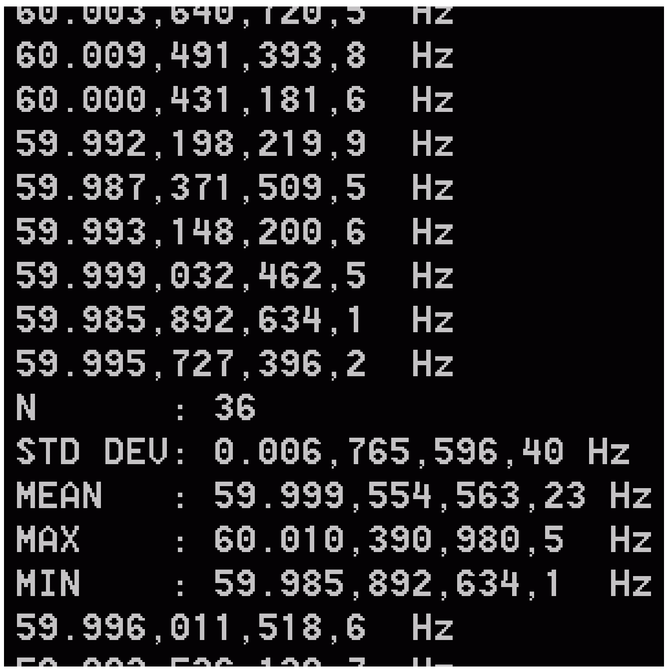
- 0.01%, 100 ppm
- Mechanical oscillator
- “Four 9’s”

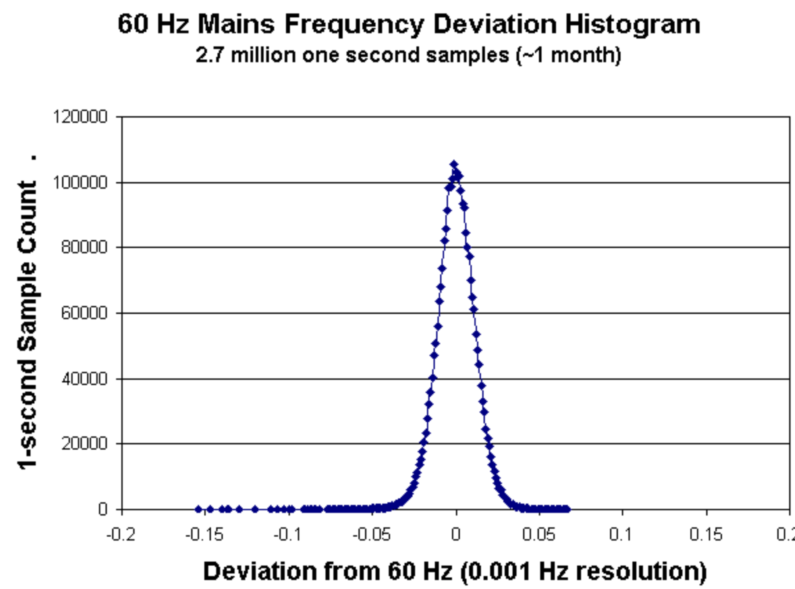




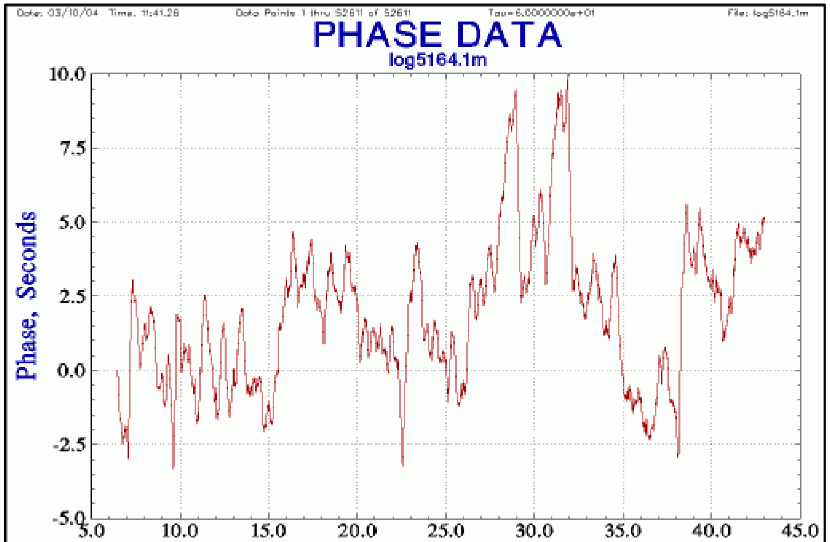
10^{-5} mains

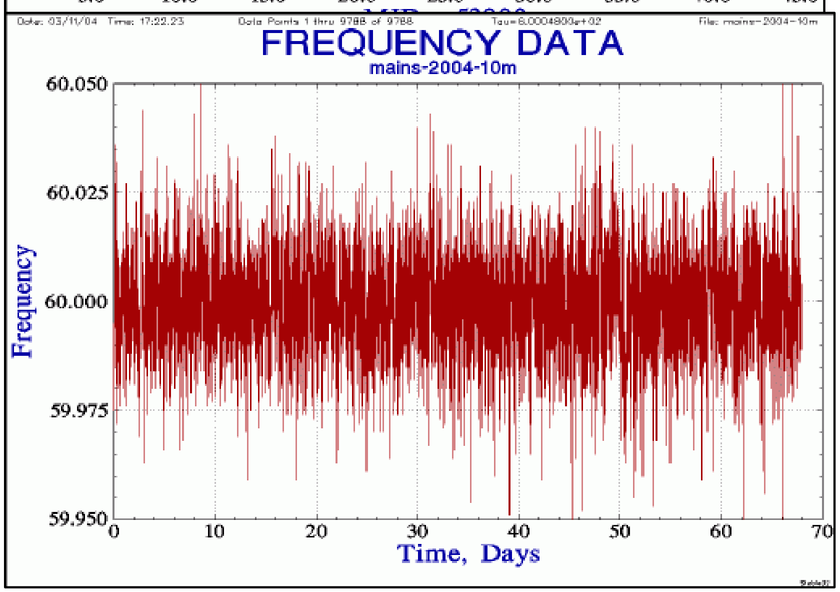
- 0.001%, 10 ppm
- 60± Hz

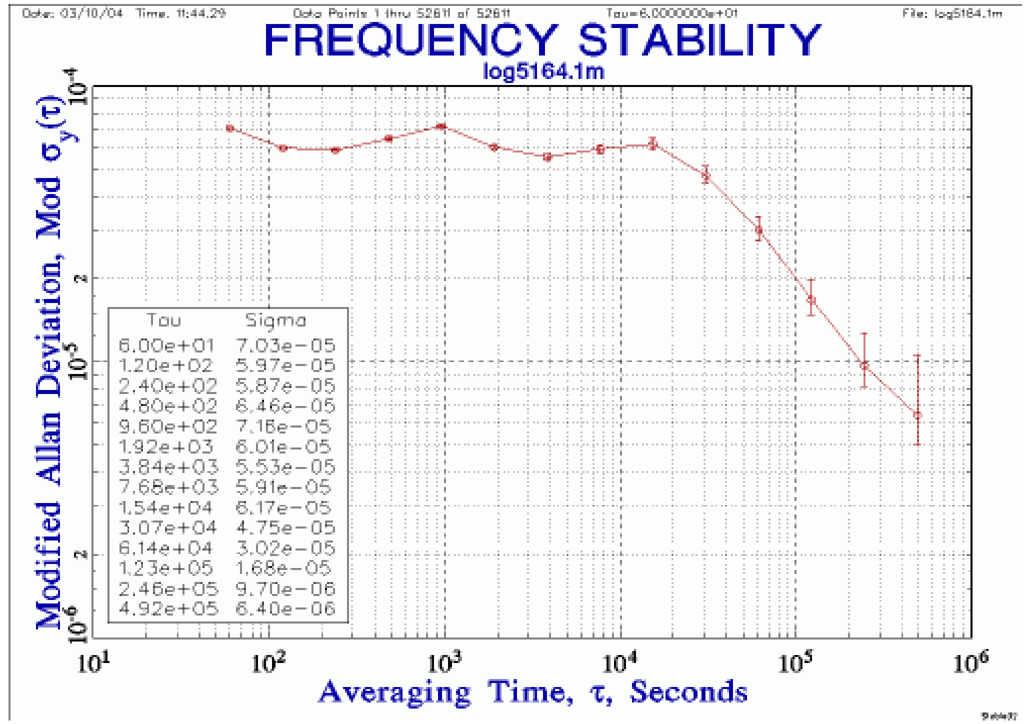




10^{-5} mains



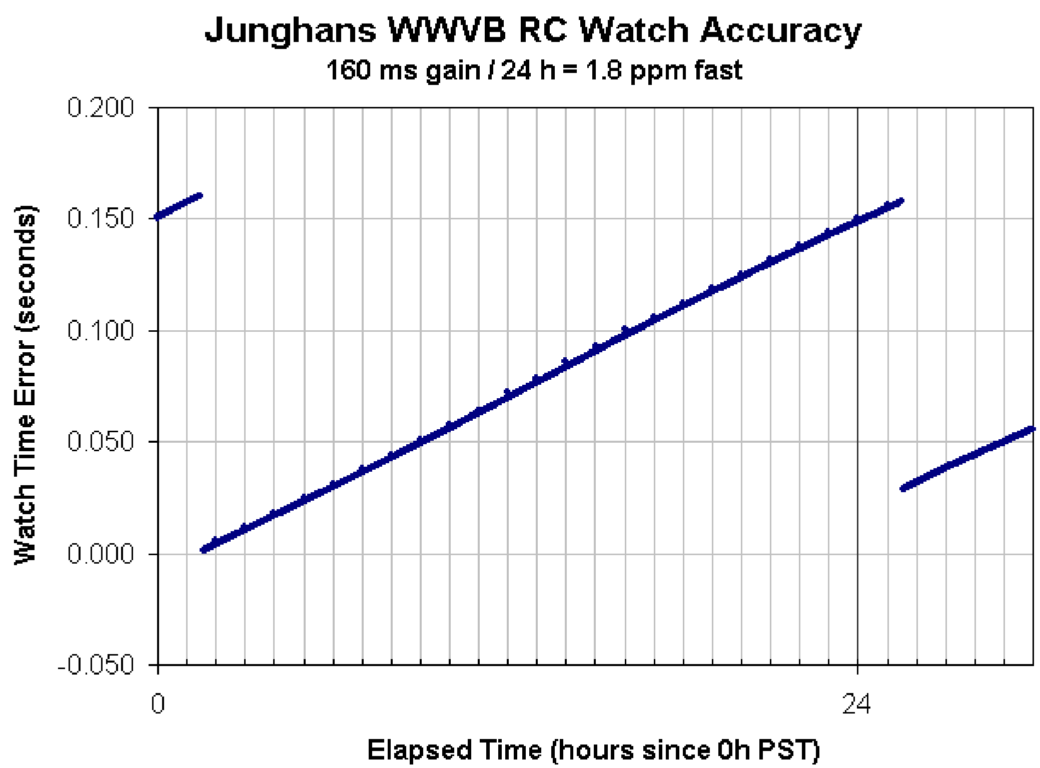




Tau	Sigma
6.00e+01	7.03e-05
1.20e+02	5.97e-05
2.40e+02	5.87e-05
4.80e+02	6.46e-05
9.60e+02	7.16e-05
1.92e+03	6.01e-05
3.84e+03	5.53e-05
7.68e+03	5.91e-05
1.54e+04	6.17e-05
3.07e+04	4.75e-05
6.14e+04	3.02e-05
1.23e+05	1.68e-05
2.46e+05	9.70e-06
4.92e+05	6.40e-06

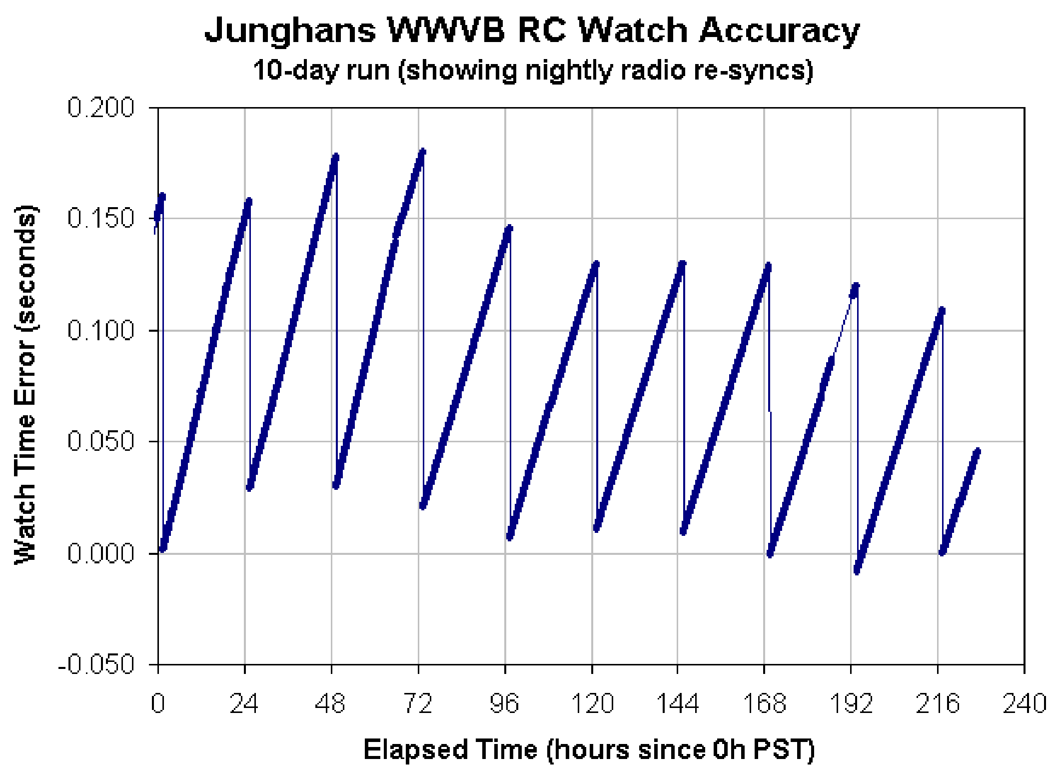
10^{-6} quartz watch (RC)

- 0.0001%, 1 PPM, 1 μs/s
- +160 ms/d = +1.85 ppm



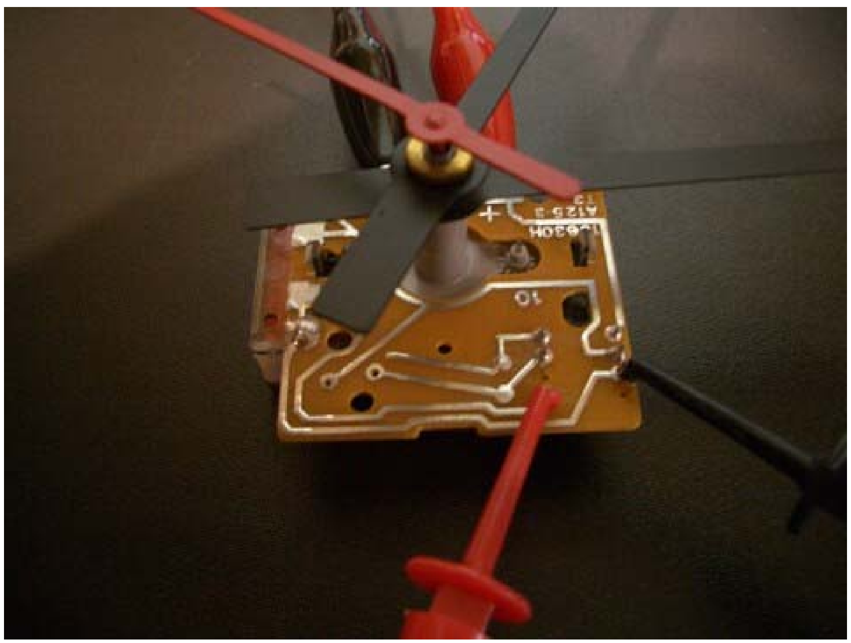
10^{-6} quartz watch (RC)

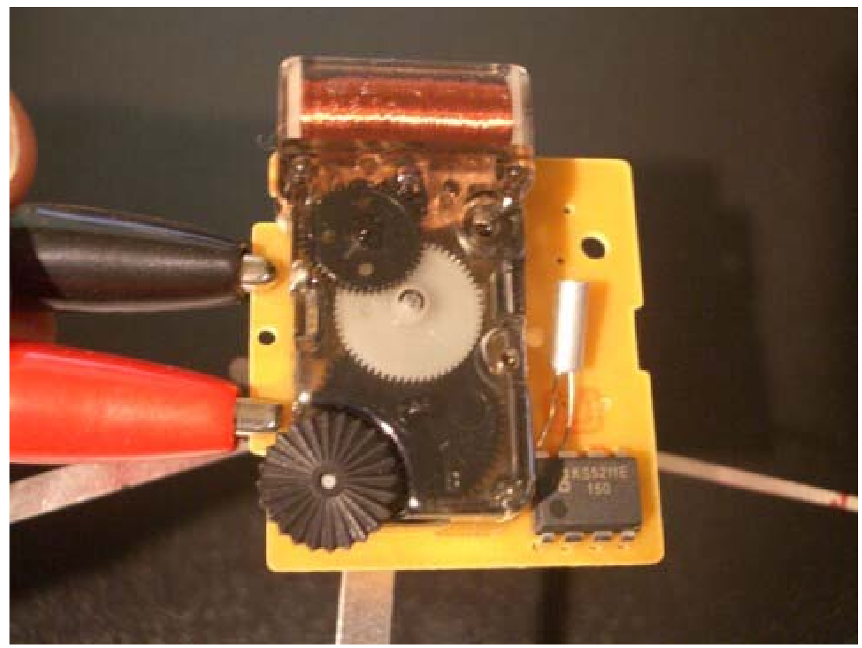
- Nightly WWVB radio sync (60 kHz)
- Look closely at 01:30 AM PST
- +1h +30m +15s
- Plot of 9 days
- Rate variations
- Sync variations



Aside: Quartz Wall Clock

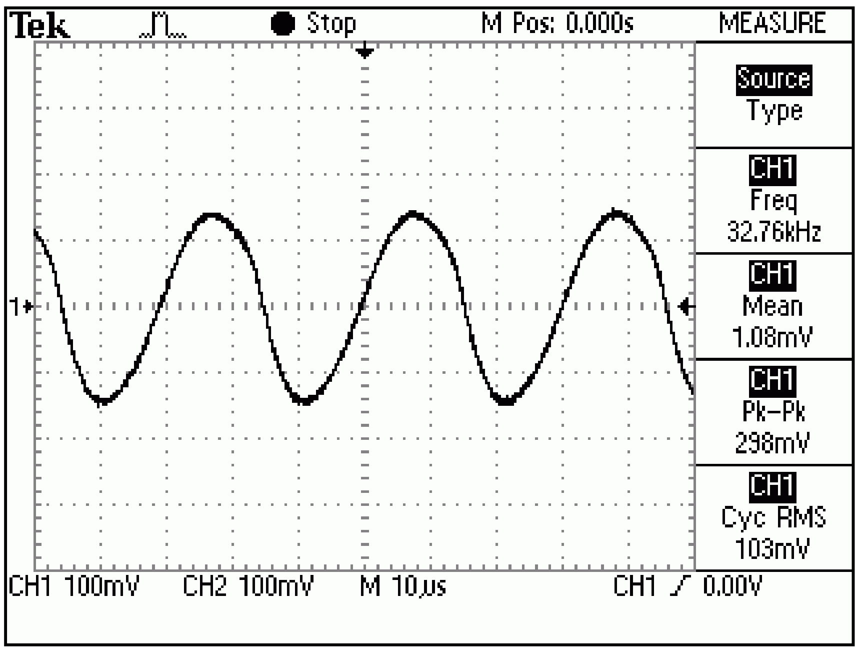
- Quartz crystal and divider/driver IC
- Stepper motor (180° per step)

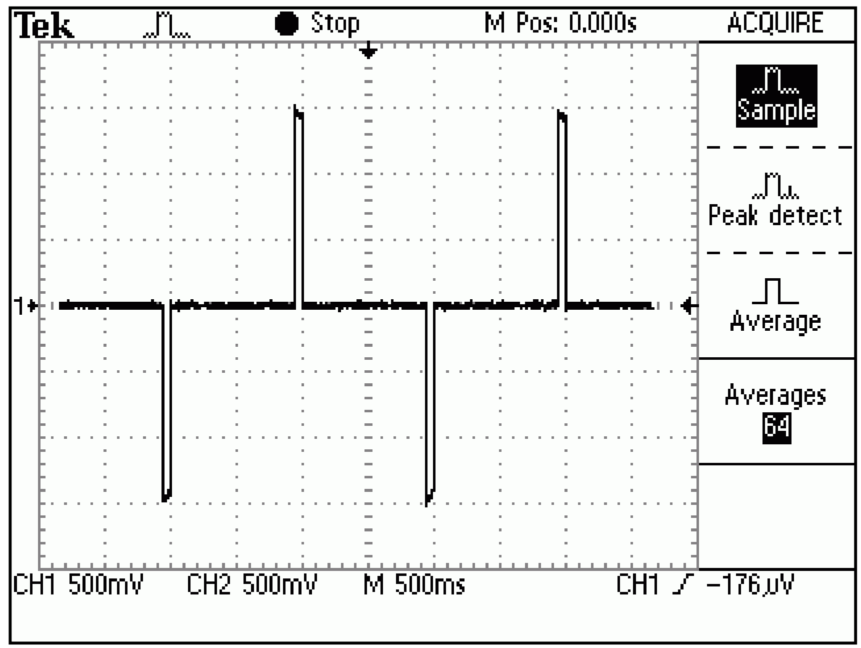




Quartz Wall Clock

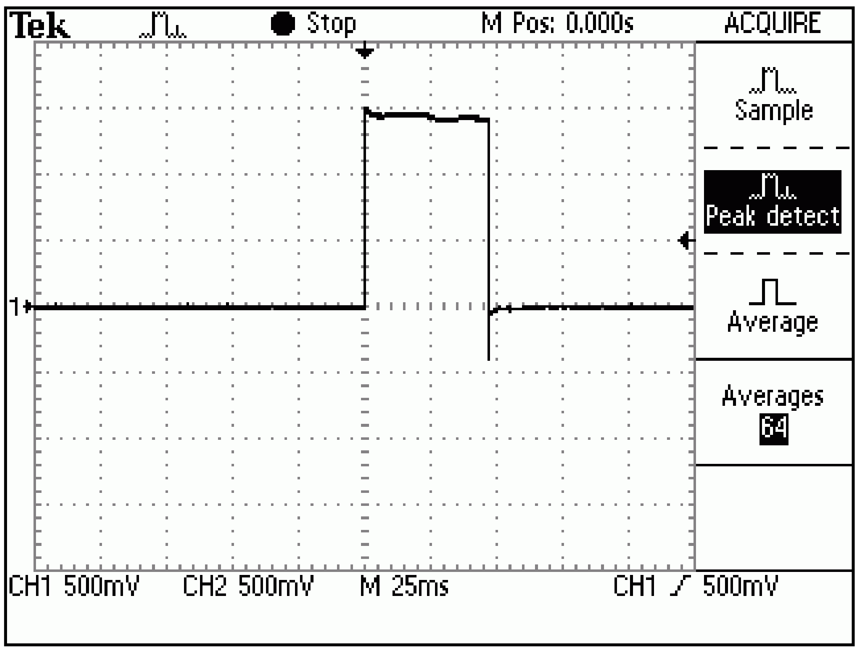
- 32 kHz oscillator
- 1 Hz stepper

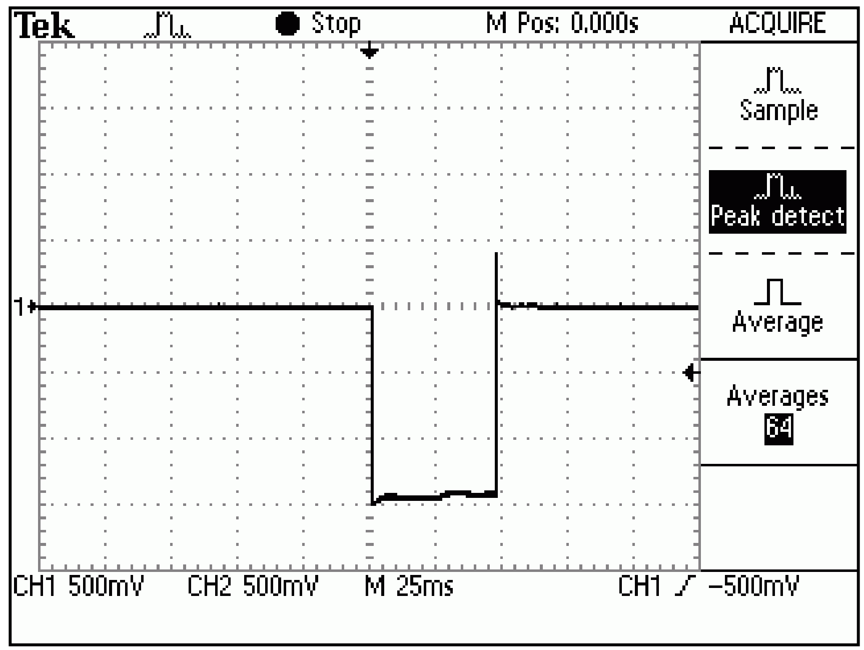




Quartz Wall Clock

- Polarity alternates
- Pulse size: 1.5 V x 50 ms





Quartz Wall Clock

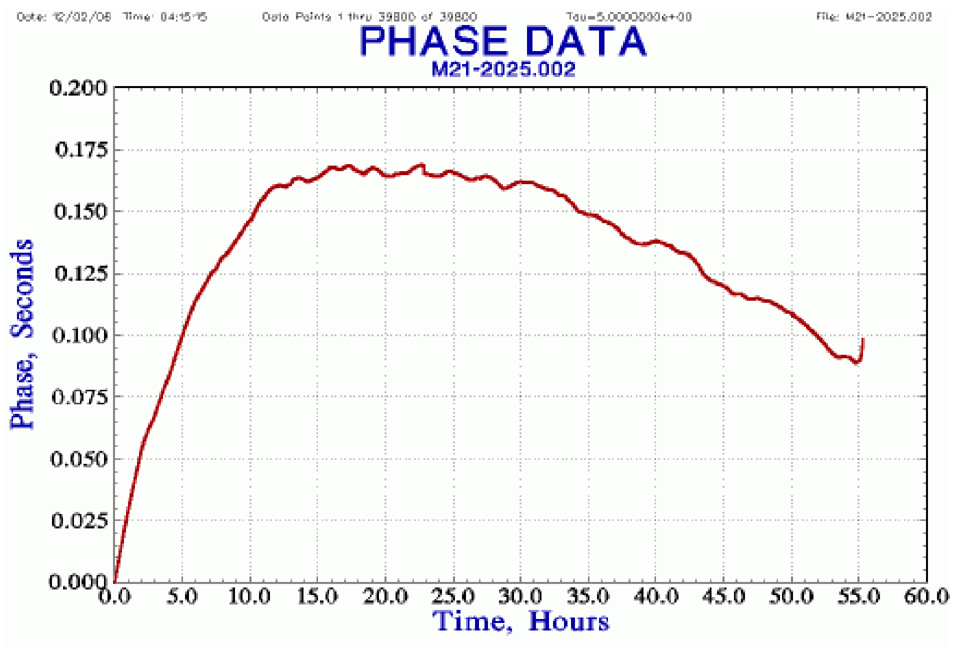
- Coil current: 1.5 V / 500 $\Omega = 3$ mA
- Oscillator current: <1 μA
- Pulse power (V x A): 4.5 mW
- Pulse width: 50 ms
- Clock Energy (P x T): 4.5 mW x 50 ms = 225 μJ = 60 pico kWh
- AA battery (2850 mAh) = ~2 years

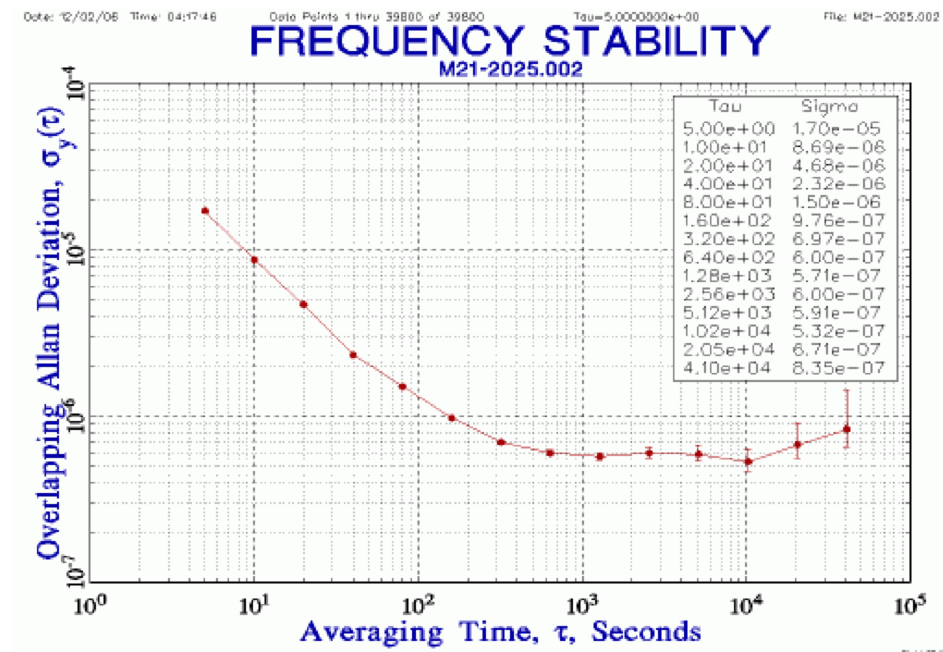
10^{-7} chronometer

- 0.1 ppm
- Rated $\frac{1}{4}$ sec/day deviation

10^{-7} chronometer

- ~55 hour runtime
- 200 ms phase residuals
- ADEV 6×10^{-7}





10^{-7} chronometer

- From 1940's USN manual…
- Phase
  - Dial error
- Frequency
  - Daily rate
- Drift
  - Deviation in rate

COMPUTATION OF RATE

Date	Dial Error + = Fast - = Slow		Daily Rate + = Gain - = Loss	Mean Deviation in Daily Rate	Remarks
	Min	Sec			
Oct 1948					
3	+0	2			Started + Set
4	+0	2½	+½		
5	+0	2½	0	¼	
6	+0	3	+½	¼	
7	+0	3	0	¼	
8	+0	3½	+½	¼	
9	—	—	—	—	Not wound
10	+0	4	+¼	—	2 day avg.

(Mean daily rate = +1/4 second)

In Table I, there will be noted a column headed "Mean Deviation in Daily Rate." The

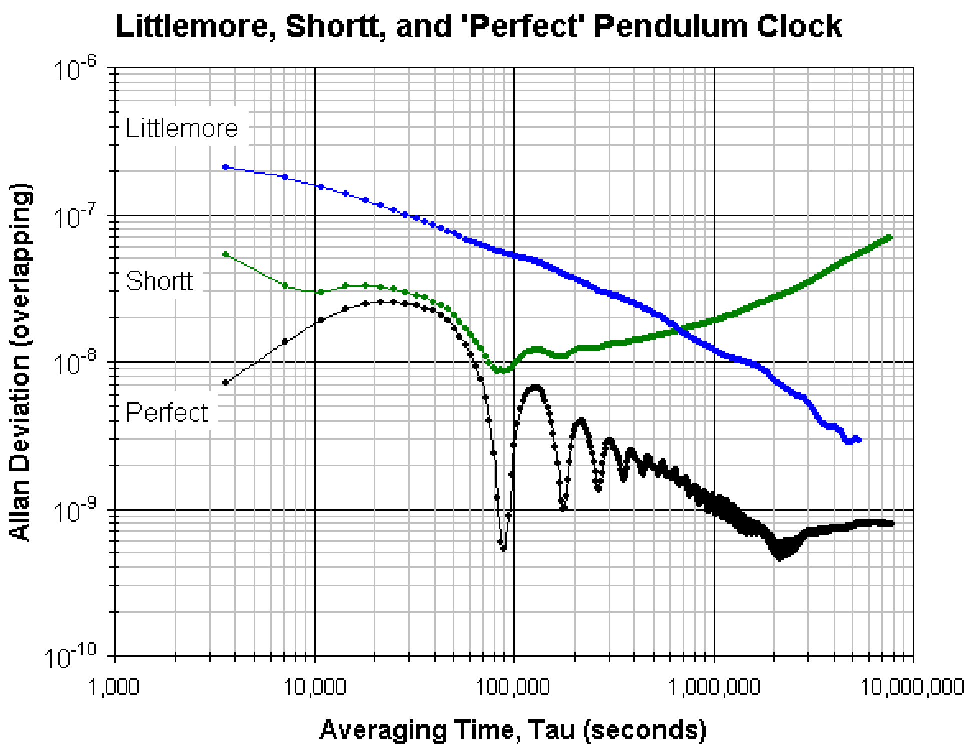
10^{-8} pendulum clock

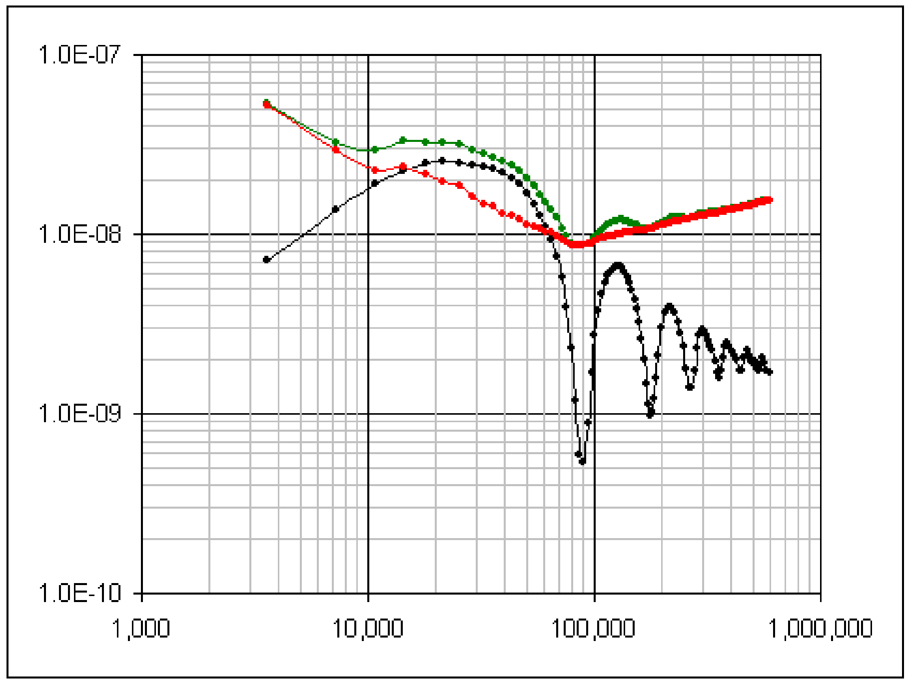
- 0.01 ppm, 10 ppb
  10 ns/s, 864 μs/d
- Shortt,
  Fedchenko,
  Riefler,
  'Littlemore'

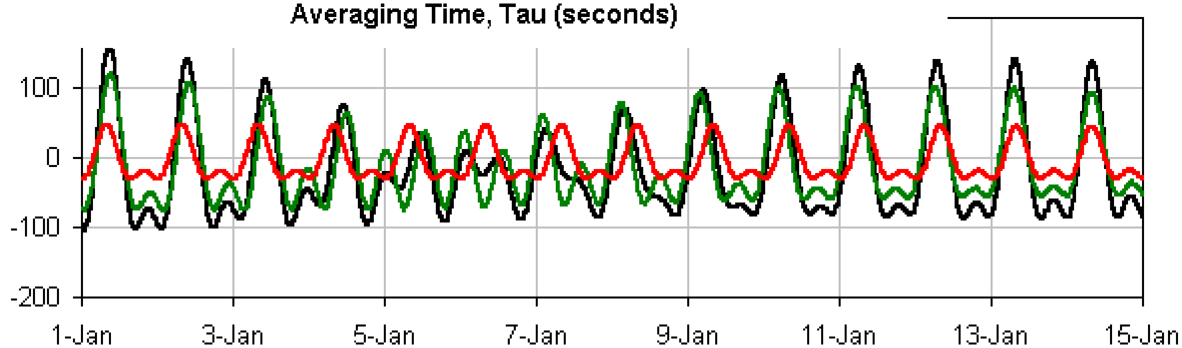
10^{-8} pendulum clock

- Amazing astronomical pendulum clocks
- Several centuries of understanding and perfection. Limitations addressed:
- Temperature, humidity, mass, friction, metallurgy, escapement, master/slave, vacuum, isochronous suspension, etc.
- When all factors solved, the best pendulum clock is just a good gravimeter

10^{-8} pendulum clock



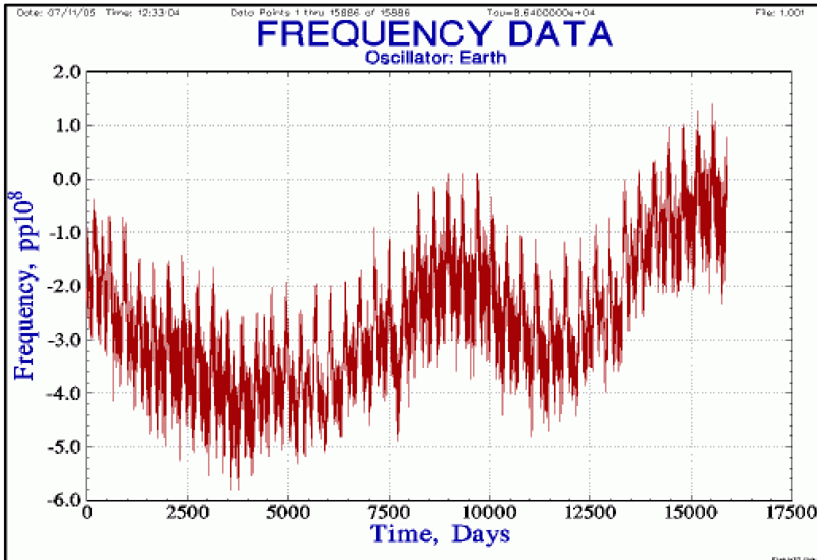


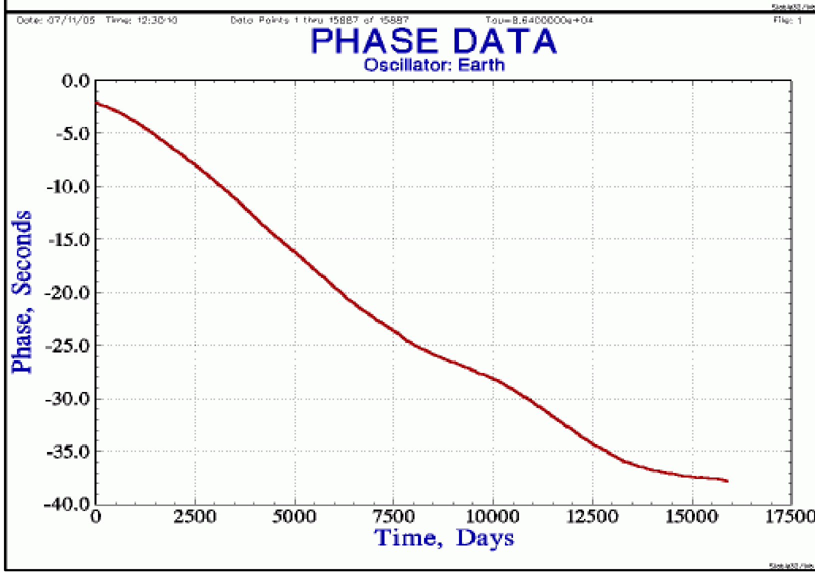


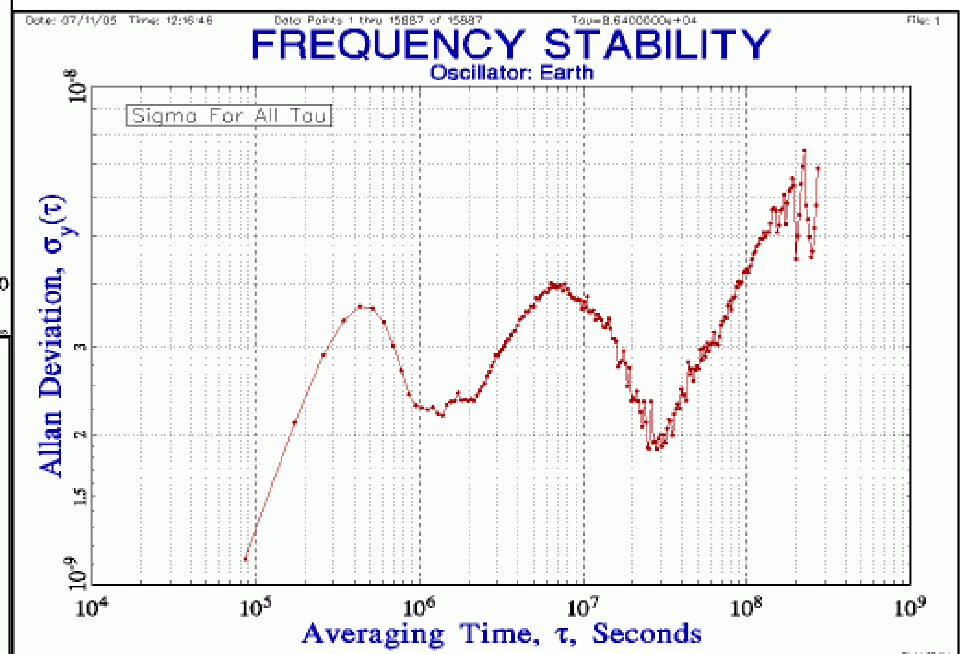
10^{-9} earth

- 0.001 ppm
- Slow by ~2 ms per day
- Also somewhat irregular
- ADEV $10^{-8} \sim 10^{-9}$
- Limited by core, weather, climate
- Also lunar/solar tides

10^{-9} earth


FREQUENCY DATA
Oscillator: Earth
Frequency, pp10^8
Time, Days


PHASE DATA
Oscillator: Earth
Phase, Seconds
Time, Days

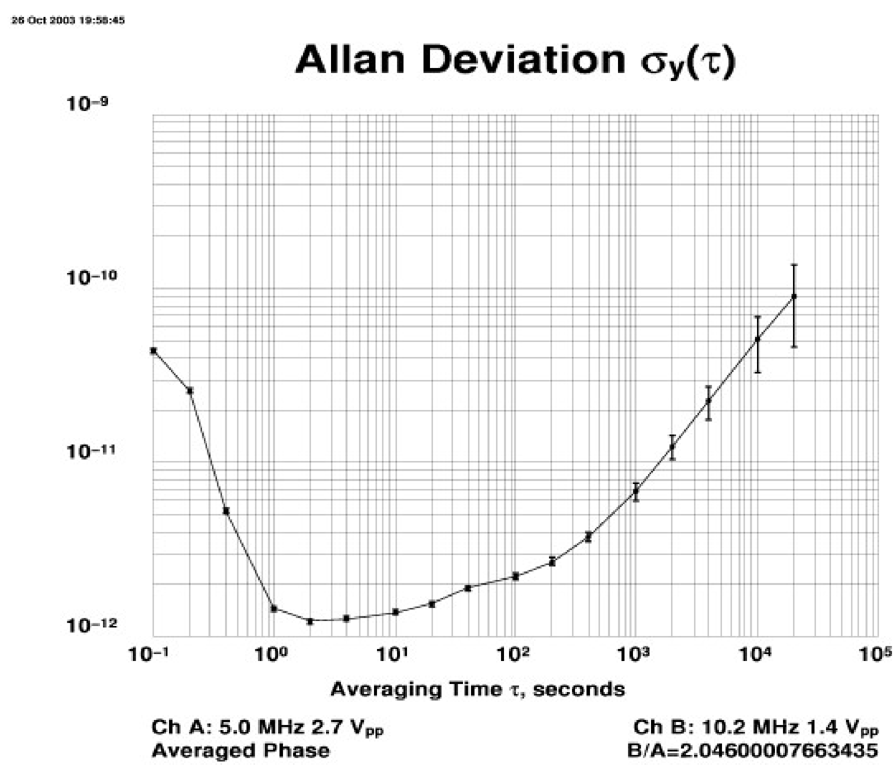

FREQUENCY STABILITY
Oscillator: Earth
Sigma For All Tau
Allan Deviation, $\sigma_y(\tau)$
Averaging Time, τ, Seconds

10^{-9} earth

- Earth as a frequency standard
- Suggested improvements:
  - Thoroughly clean, and dry with cloth
  - Remove surrounding gas and water vapor
  - Wait for core to cool before use
  - Re-align axis of rotation (wobbling)
  - Keep away from nearby moon (tides)
  - Keep away from sun (tempco)
  - Re-adjust rate (avoid leap seconds)

10^{-10} ocxo

- 0.1 ppb, 100 ps/s, 8.64 μs/d
- 10^{-10}…10^{-13} short
- 5×10^{-10}/d drift



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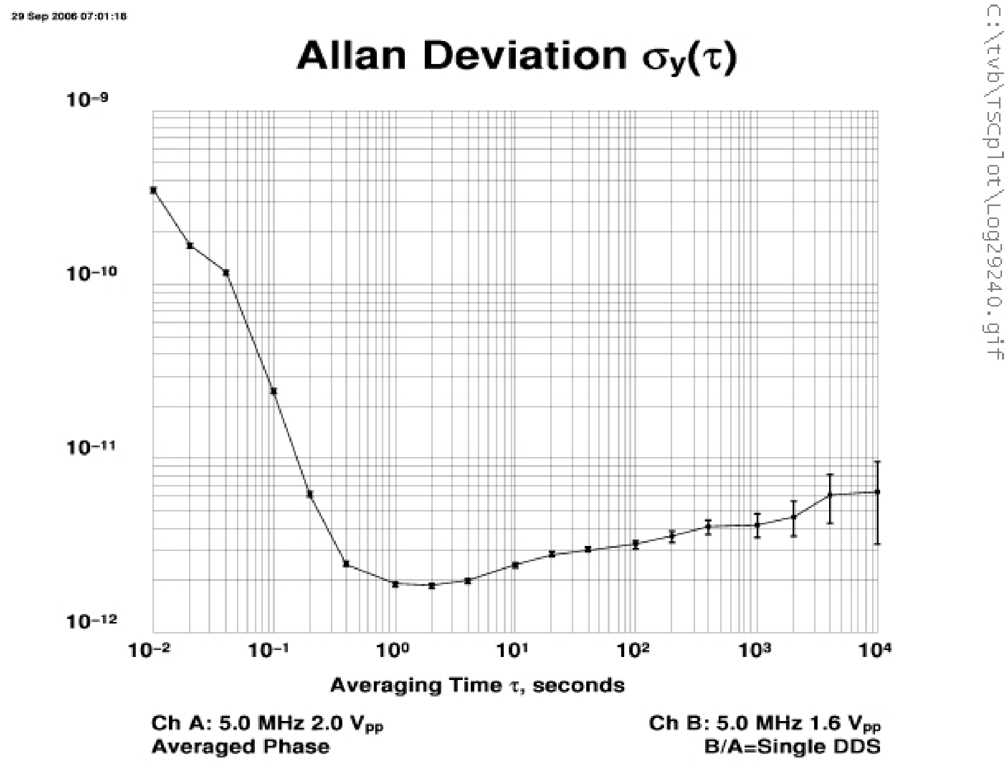
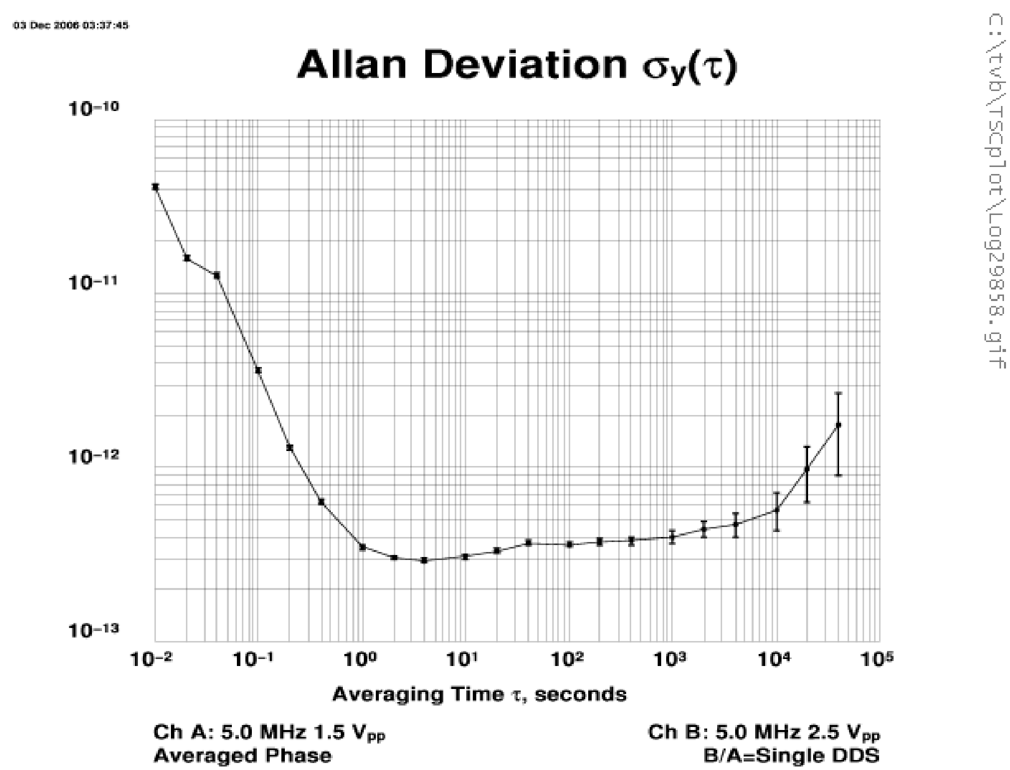
10^{-11} good ocxo

- 0.01 ppb, 10 ps/s, 864 ns/d (~1 μs/d)
- 10^{-11}…10^{-13} short
- ~10^{-11}/d drift



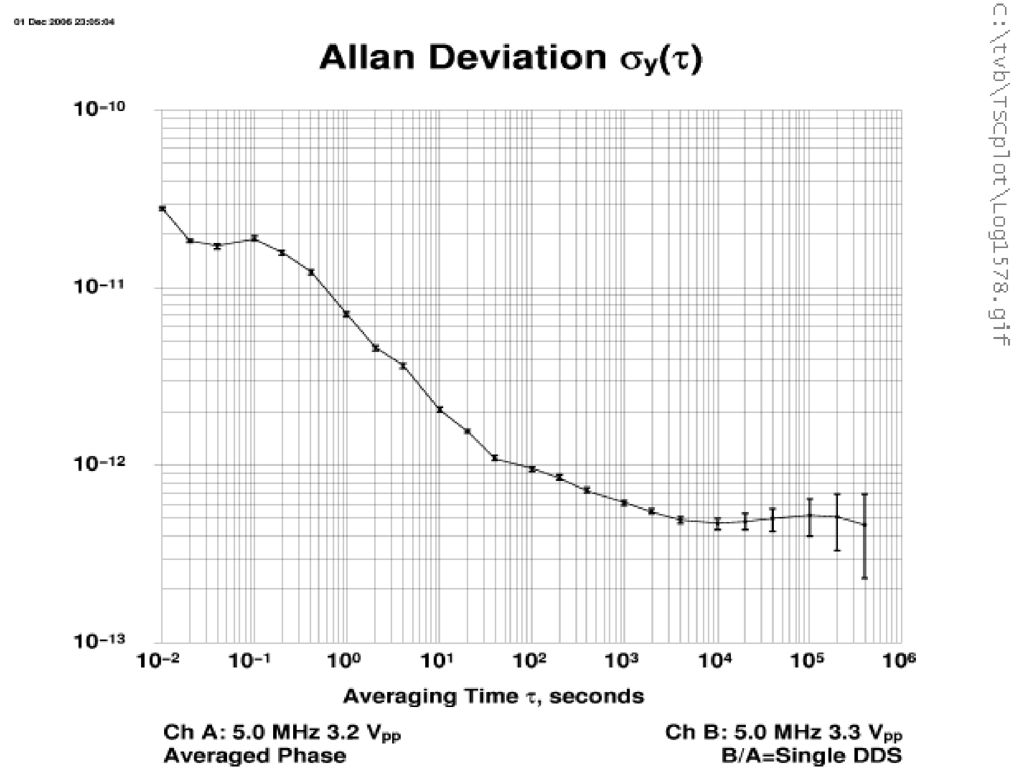
10^{-12} excellent ocxo

- 1 ppt, 1 ps/s, 86.4 ns/d (~100 ns/d)
- $\sim 10^{-13}$ short/mid
- $\sim 3\times 10^{-12}$/d drift



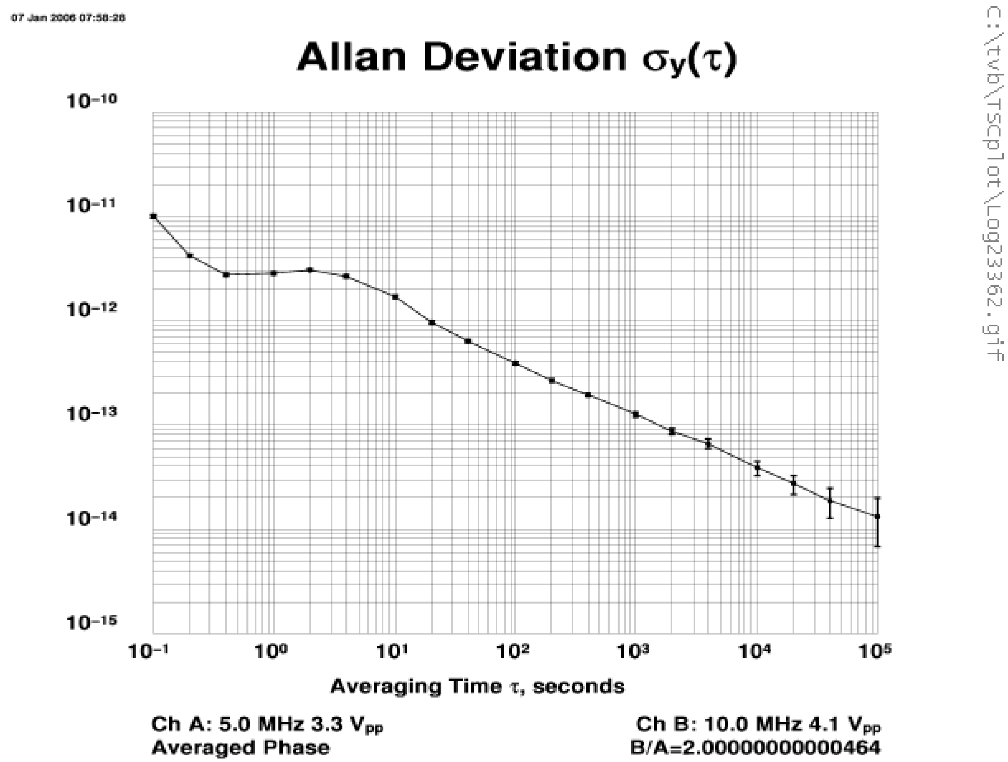
10^{-13} rubidium

- 8.64 ns/d (~10 ns/d)
- ~10^{-13} mid-term
- ~1×10^{-11}/m drift



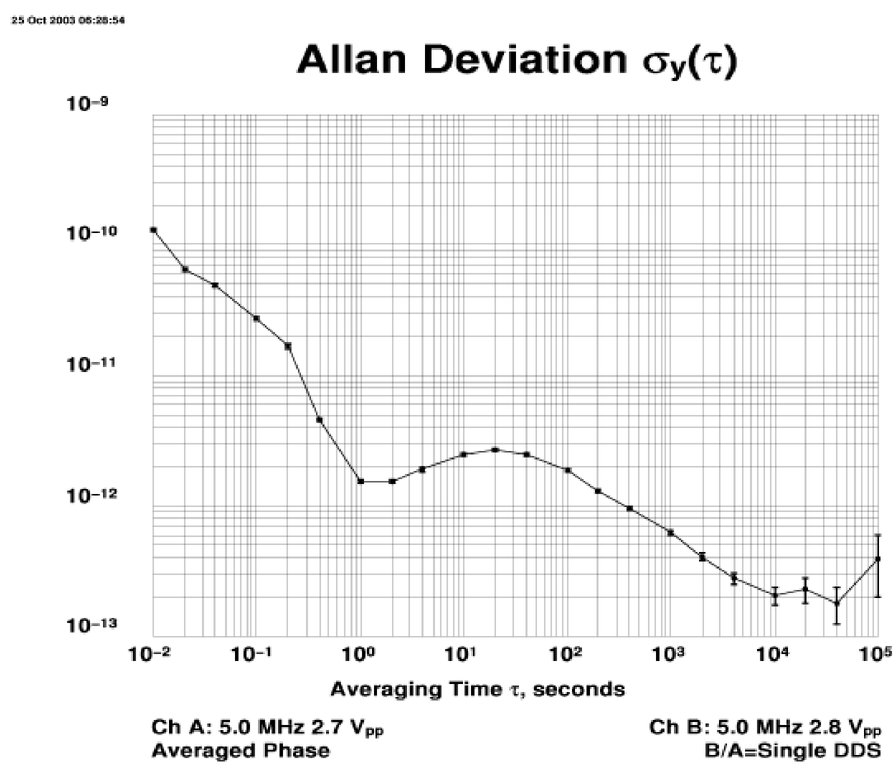
10^{-14} cesium

- 864 ps/d (~1 ns/d)
- ~10^{-13} mid-term
- ~1×10^{-14} @ 1 day



10^{-14} more cesium

- 10^{-14} not!
- Cesium clocks differ by 2x – 50x
- E.g., old 5060A vs. new 5071A



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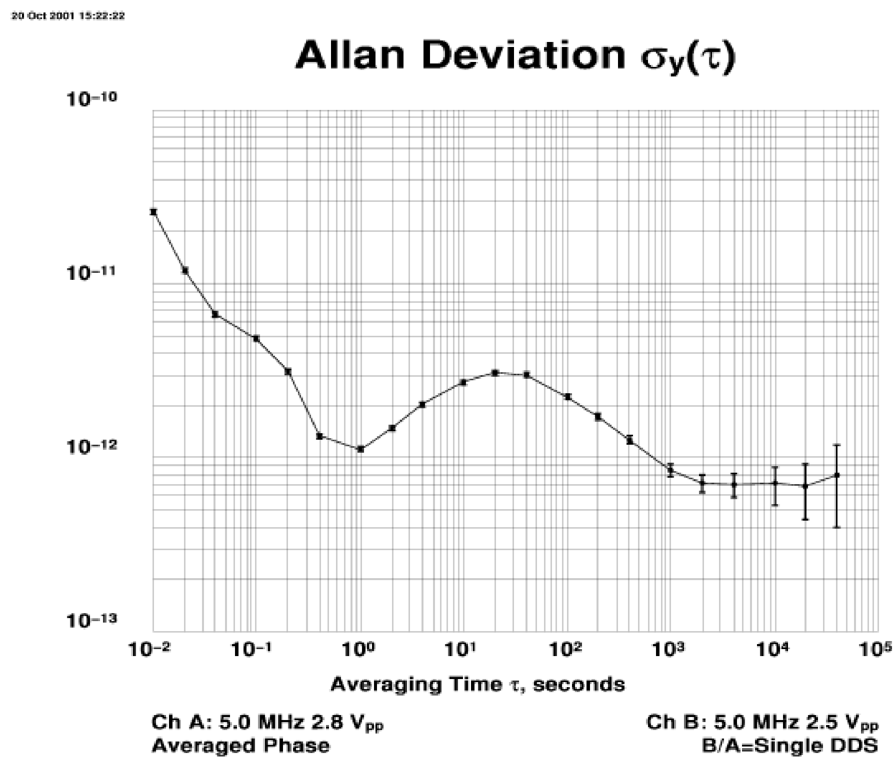
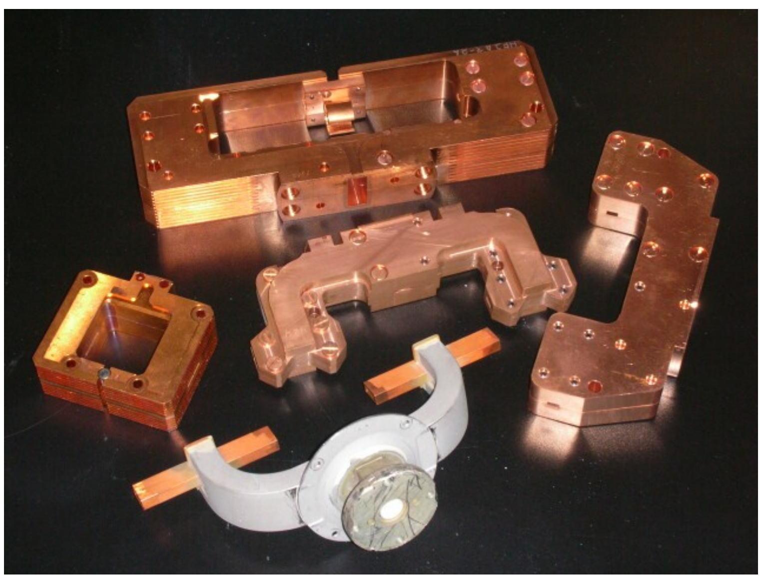
10^{-14} another cesium

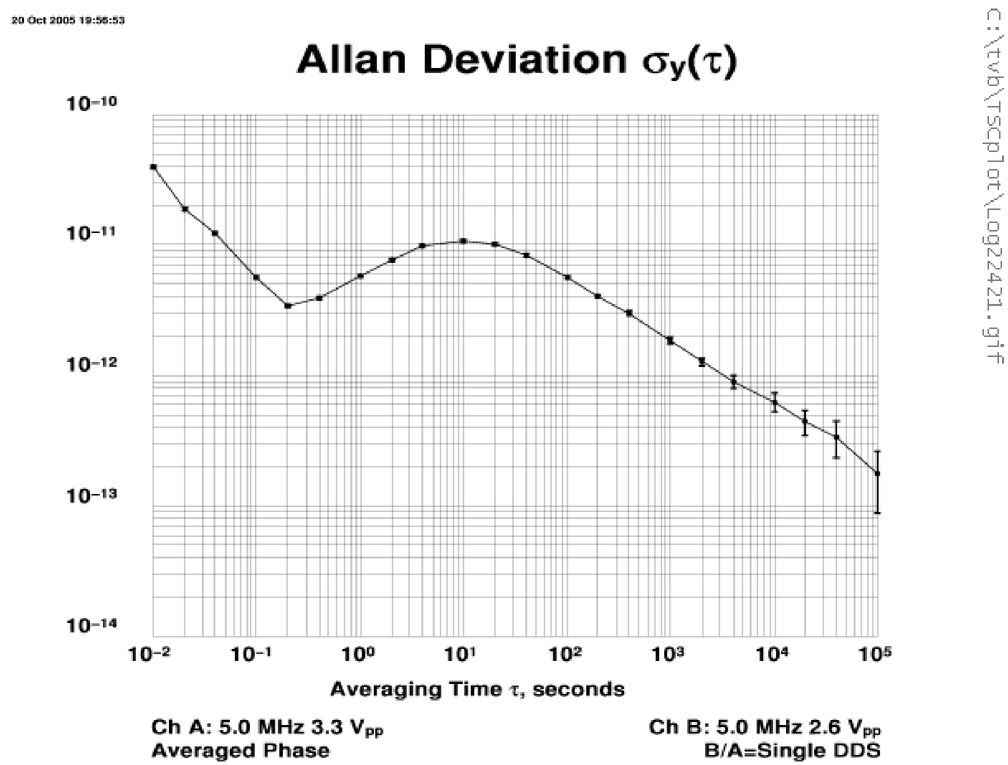
- Not even close to 10^{-14} @ 1 day
- FTS 4010
- Portable Clock
- Old, tired



10^{-14} one more cesium

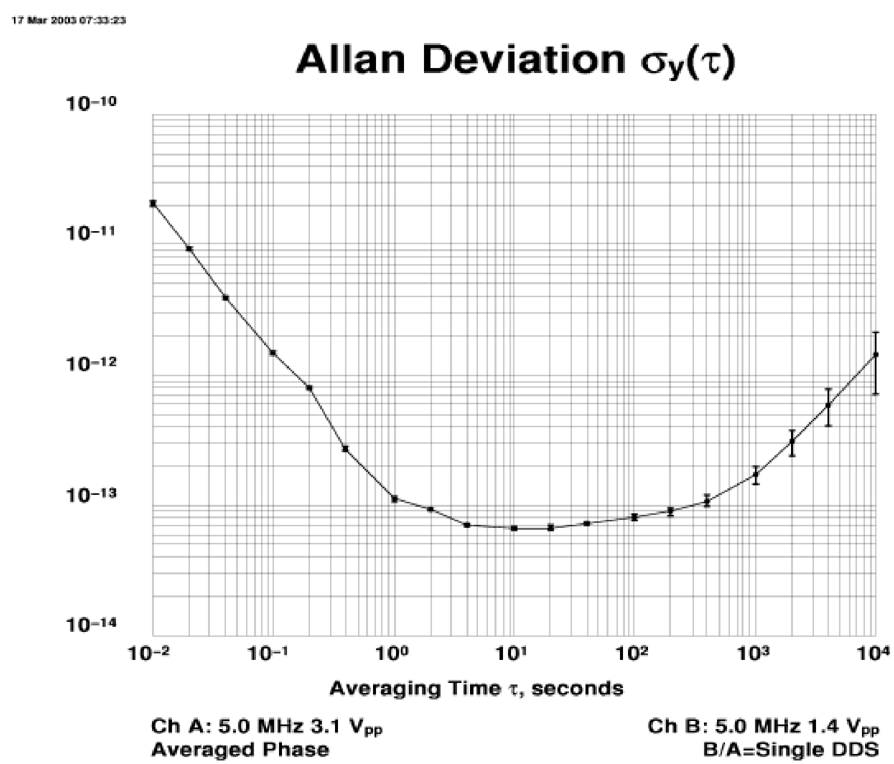
- About 2×10^{-13} @ 1 day
- FTS 4050
- See variety of Cs beam cavities





10^{-14} BVA quartz

- But can you get to high altitude and measure in 1 to 100 seconds?
- $10^{-13} \ldots 10^{-14}$ short
- $10^{-11} \ldots 10^{-12}$ drift



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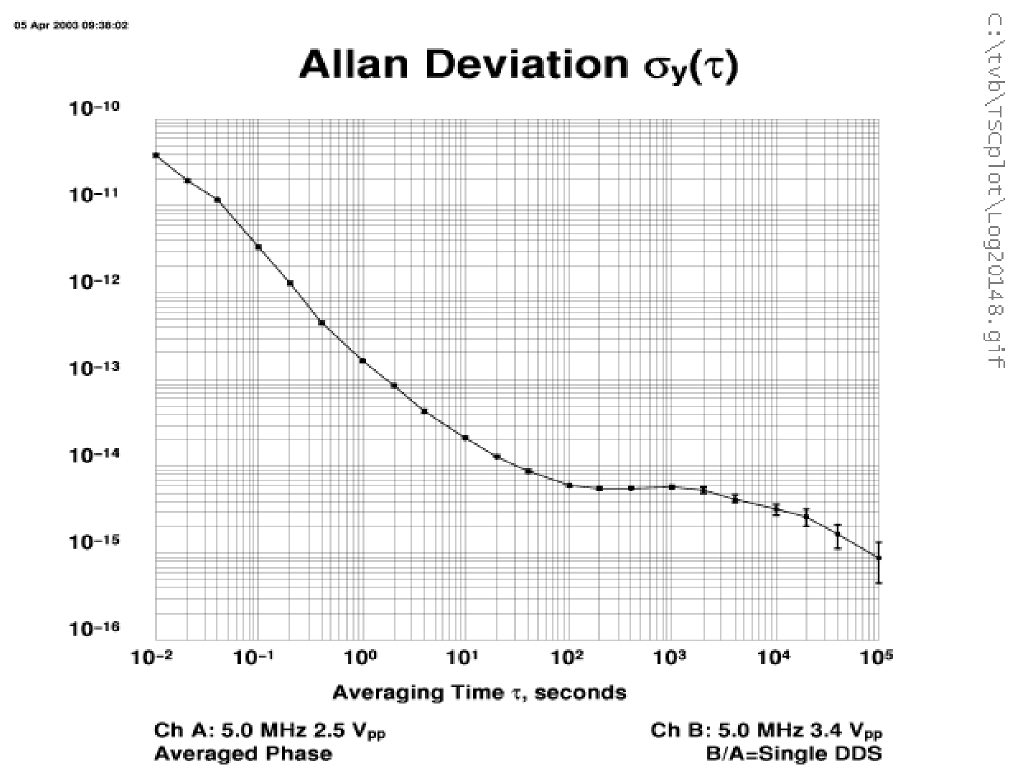
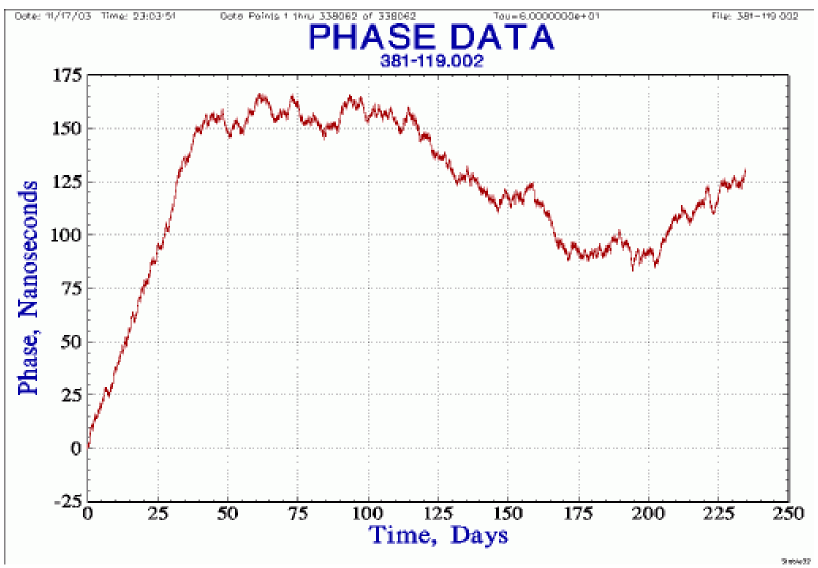
10^{-15} active h-maser

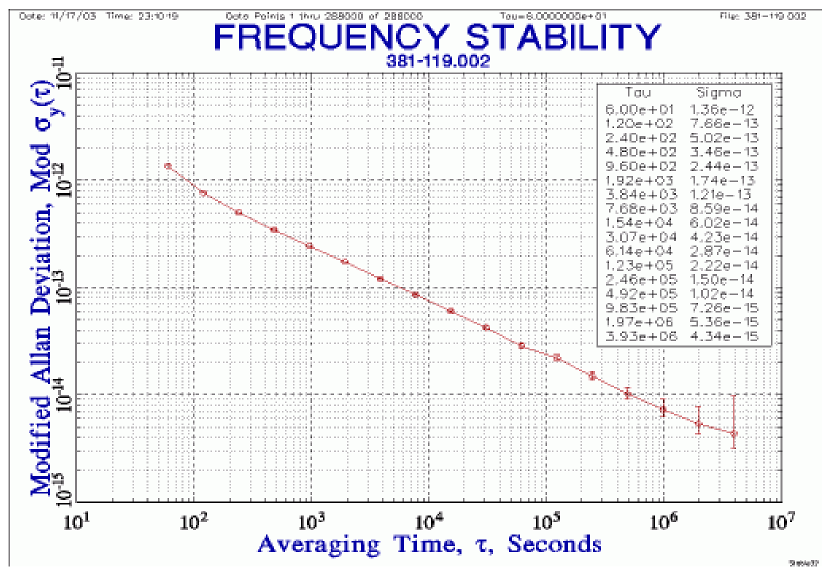
- 86.4 ps/d
- Near 1×10^{-15} @1d
- Cavity auto-tuned



10^{-15} cesium, long-term

- High-perf units
- Pair near 2×10^{-14} at a day
- Floor near 5×10^{-15} in weeks





Powers of Ten – summary

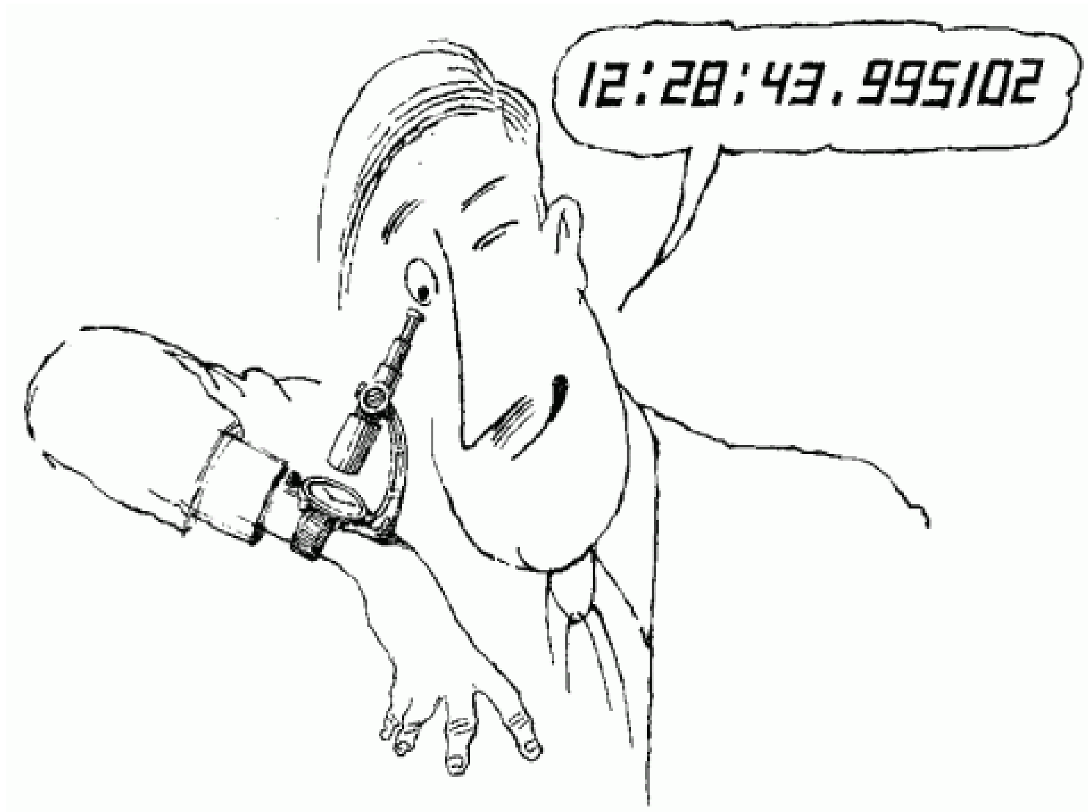
- 10% to 10^{-15} – 15 orders of magnitude

Chapter 6

- Experimental setup

Key Parameters

- How high
- How long
- How stable
- How many
- How precise you measure



Cartoon by Dusan Petricic
Scientific American column Wonders by Philip and Phyllis Morrison
http://www.sciam.com/1998/0298issue/0298wonders.html

Ingredients

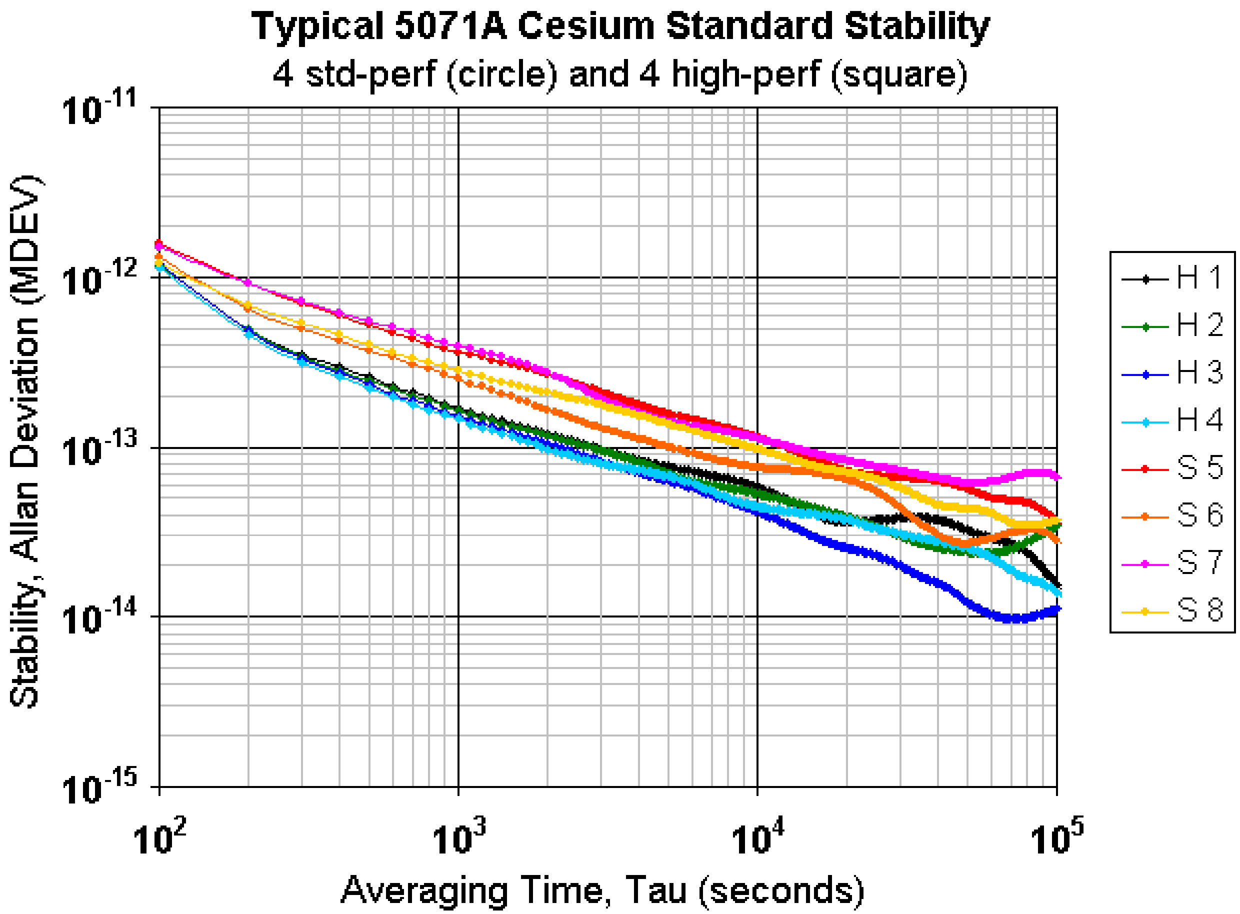
- Portable clocks, *hp* 5071A, @3
  - Check against base clock (home)
  - Check against themselves (away)
- Base clock (passive H-maser)
  - Check against active H-maser)
  - Check against GPS @2
- Measurement system
  - *hp* 53132A TIC (~150 ps) @3
  - Laptop

Portable Clock(s)

- Many Cs tested (5060,5061,4050,4060)
- Three good 5071A/001 chosen
- Color coded clocks, cables, TIC, kids
- Deliberate phase offset of +1, +2, +4 μs
- 1 PPS cables kept with clocks
- HP 53132A TIC per clock
- Data logged at 1 Hz; 5 min averages

5071A

- Pick best ones…



Data collection

- RS232 interface
- PC serial port logger
- MJD time-stamps
- 8 port serial-to-USB
- 53132A ti,mean,sdev
- 5071A stats
- GPS NMEA

```
53630.228067 5.775,8  us
53630.228067 N      : 300
53630.228067 STD DEV: 0.000,390 us
53630.228067 MEAN   : 5.775,600 us
53630.228067 MAX    : 5.776,8   us
53630.228067 MIN    : 5.774,3   us
53630.228079 5.775,8  us
53630.228090 5.775,8  us
53630.228102 5.775,3  us
53630.228113 5.775,8  us
53630.228125 5.776,3  us
```

```
scpi > syst:print?

MJD     53627 06:28:28
CBT ID: US31281148(H)
Status summary: Operating normally
Power source: AC
Log status: 20  entries

Freq offset:          0e-15   Osc. control:      -8.40 %
RF amplitude 1:     30.2 %    RF amplitude 2:     28.6 %
Zeeman Freq:       39949 Hz   C-field curr:     12.204 mA
E-multiplier:       1226 V    Signal Gain:        14.4 %

CBT Oven:            7.6 V    CBT Oven Err:       0.02 C
Osc. Oven:          -9.0 V    Ion Pump:            0.0 uA
HW Ionizer:          1.0 V    Mass spec:          12.8 V
SAW Tuning:          0.0 V    DRO Tuning:          6.3 V
87MHz PLL:           0.9 V    uP Clock PLL:        3.0 V
+12V supply:        12.3 V    -12V supply:       -12.3 V
+5V  supply:         5.5 V    Thermometer:        41.1 C
```

```
53630.940405 $GPGLL,4733.2630,N,12208.3683,W,223410,A,A*50
53630.940405 $GPBOD,,T,,M,,*47
53630.940405 $PGRME,10.6,M,13.2,M,17.0,M*1F
53630.940405 $PGRMZ,980,f,3*1A
53630.940405 $GPRTE,1,1,c,*37
53630.940428 $GPRMC,223412,A,4733.2630,N,12208.3683,W,0.0,0.0,170905,18.1,E,A*3C
53630.940428 $GPRMB,A,,,,,,,,,,,,A,A*0B
53630.940428 $GPGGA,223412,4733.2630,N,12208.3683,W,1,04,2.9,298.4,M,-18.4,M,,*7F
53630.940428 $GPGSA,A,3,,08,,,,,19,,27,28,,,5.1,2.9,4.3*35
53630.940428 $GPGSV,3,1,11,07,21,191,36,08,76,089,49,10,07,249,00,11,14,099,00*7A
53630.940428 $GPGSV,3,2,11,13,00,161,00,18,00,347,00,19,23,044,42,26,35,309,00*72
53630.940428 $GPGSV,3,3,11,27,44,114,44,28,77,263,45,29,45,297,00*42
```

Base Clock, 'House' standard

- Master 1 PPS, CH1-76 passive H-maser
- 1PPS distribution system (8x)
- 1PPS vs. HP 58503B GPSDO
- 1PPS vs. CNS-II (sawtooth-less M12+)
- 1PPS used as ref for 5071A (x3)
- Deliberate phase offset of -4 μs
- 5 MHz vs. active maser

AC / DC power

- Engine tap from minivan 12 VDC
- 2 batteries in parallel for 12 V
- 4 batteries in series/parallel for 24 V
- 12VDC/120VAC inverters
- AC power for clocks
- AC power for laptop(+batt) and TIC
- Triple backup for clocks (AC,DC,+batt)
- Power, voltage, and current monitors

Comment on Clock Accuracy

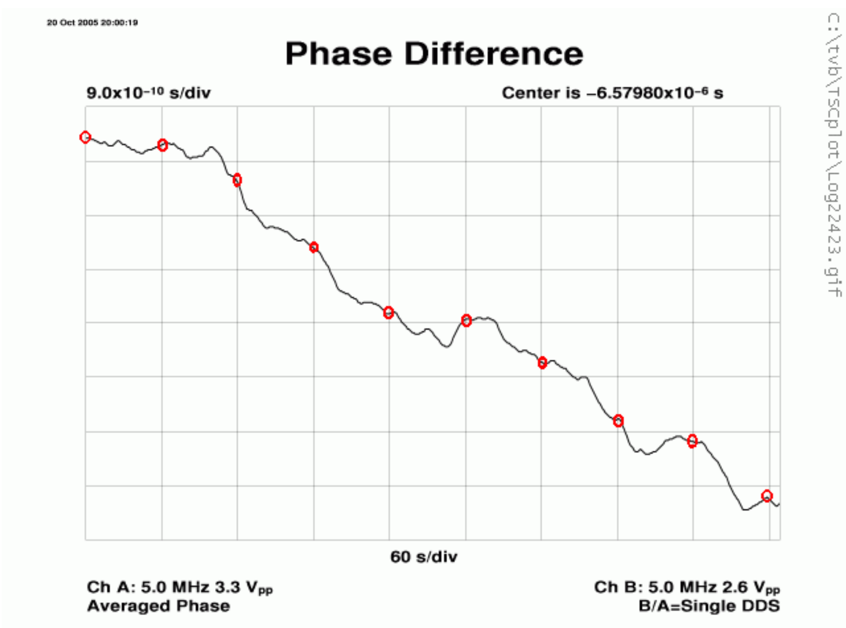
- Need to predict time 2 days in future
- Not accuracy, but stability
  - ADEV (τ=2d)
- 5071A/001 is ~1×10^{-14}
- Three clocks:
  - Redundancy (single point of failure)
  - Self-checking (one clock, two clocks…)
  - Lower uncertainty ($\sqrt{3}$ = 1.7x better)

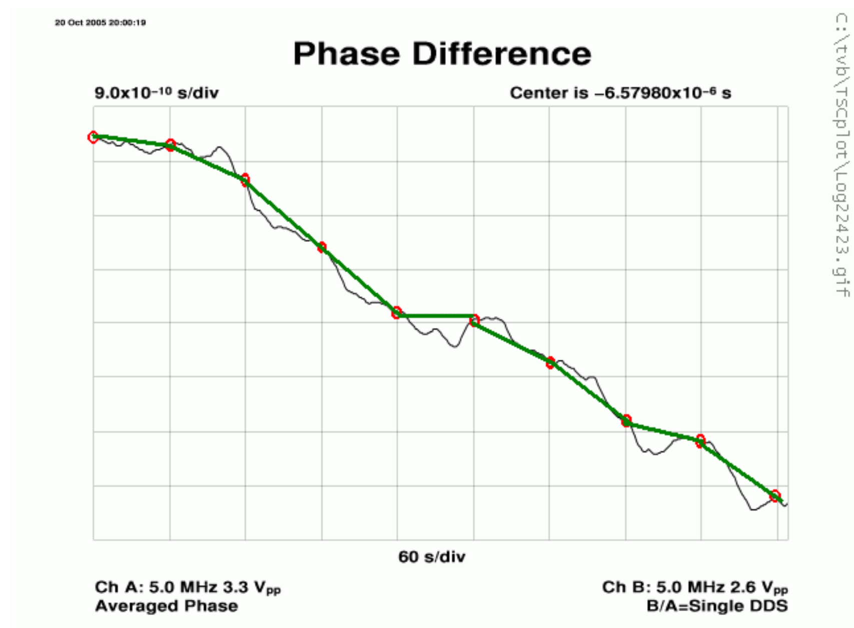
Carrying the Time

- How to measure clocks when at home?
  - Compare with house reference
  - Compare amongst themselves
- How much to trust clocks when away?
  - What do clocks do when you're not looking?
  - Guess future behavior = known past
  - Past statistics give future predictions
  - 'Certainty' replaced with 'confidence'

Stability: Measurements

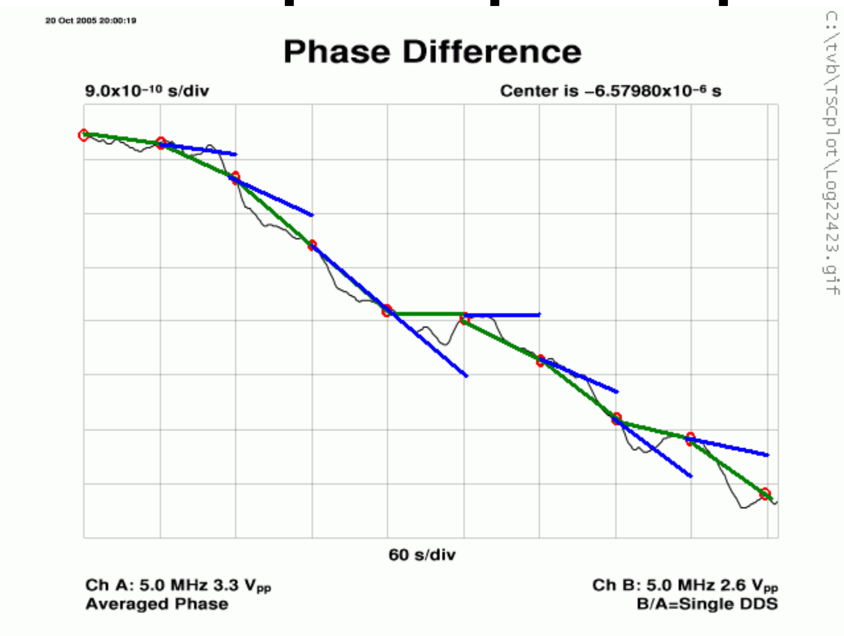
- Phase **measurements**: p0, p1, p2, …
- Frequency *calculations* (tau T): f1 = (p1 – p0)/T, f2 = (p2 – p1)/T, …

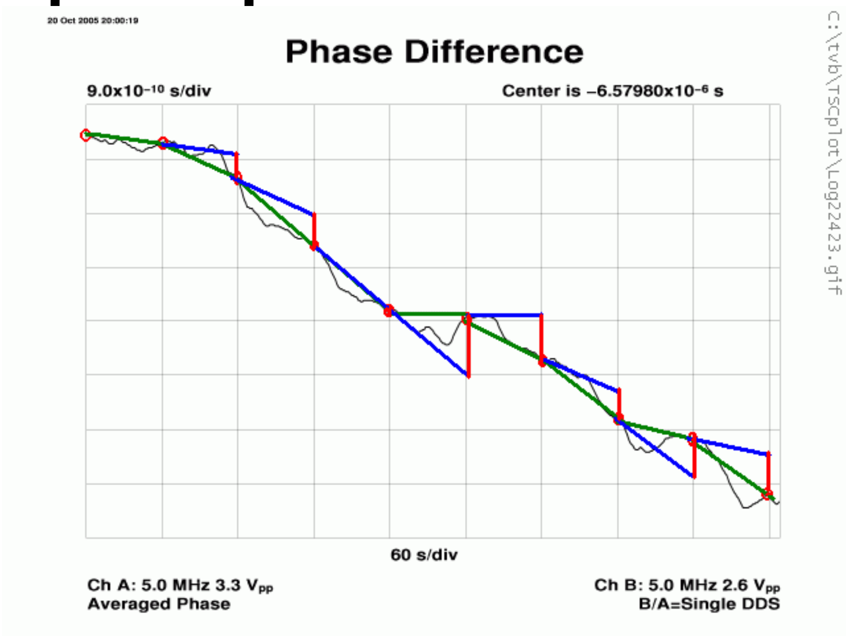




Stability: Predictions

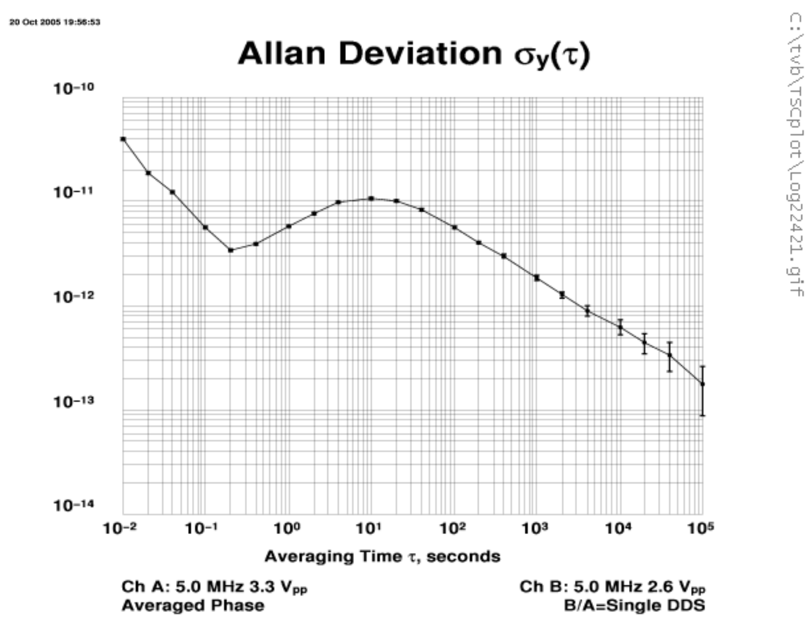
- Predict (extrapolate) future phase:
  $p2' = p1 + f1 \cdot T = p1 + (p1 - p0)$
- Later, check prediction error:
  $e2 = p2 - p2' = p2 - 2p1 + p0$





Stability: Allan Deviation

- Stddev of prediction errors gives you estimate of future accuracy of clock
- This Σ[p2 – 2p1 + p0] thing is ADEV



20 Oct 2005 19:58:12

Allan Deviation Table

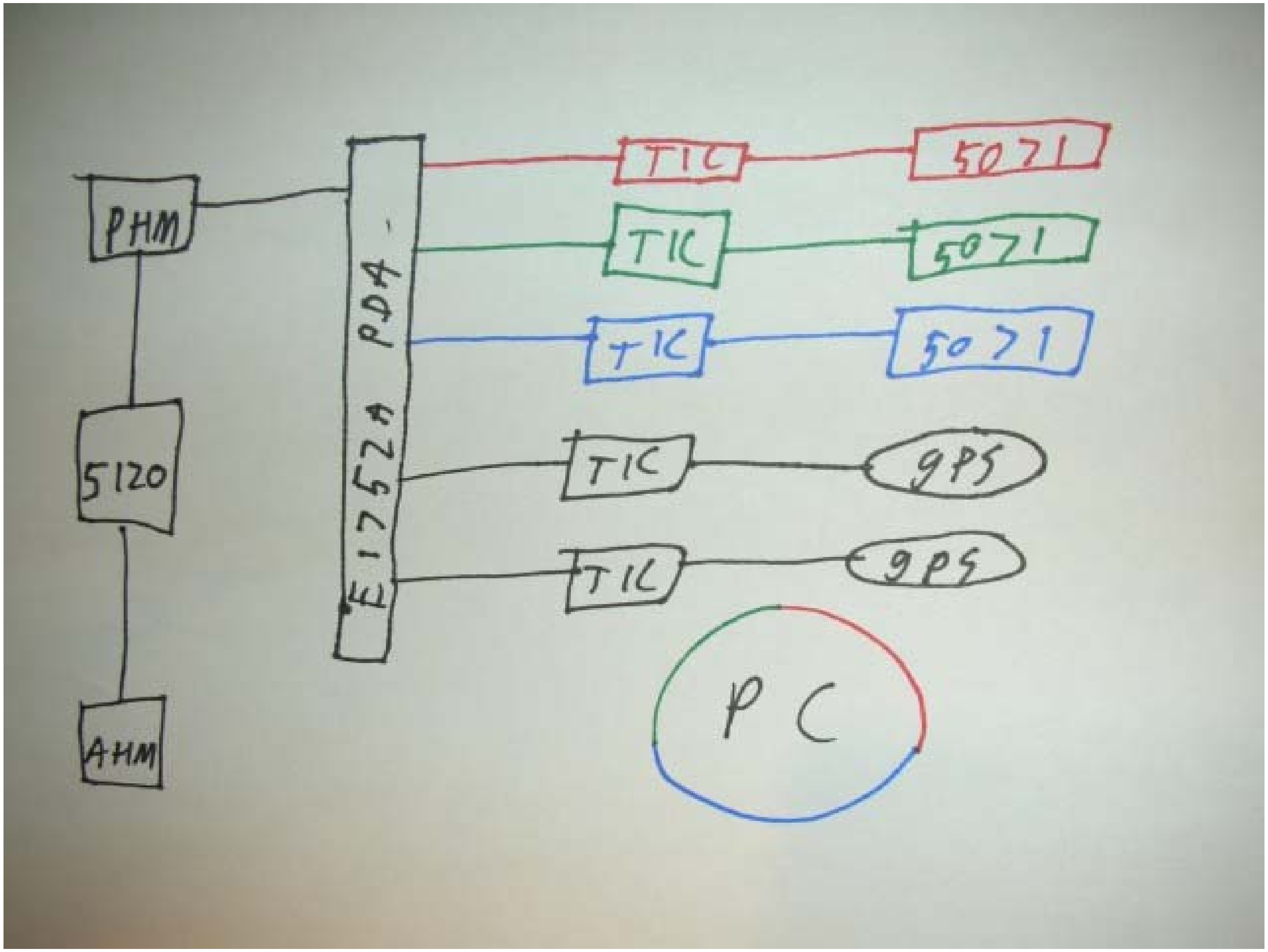
Avg. Time (s)	Allan Deviation $\sigma_y(\tau)$	Avg. Time (s)	Allan Deviation $\sigma_y(\tau)$
0.01	$3.99 \pm .12 \times 10^{-11}$	400	$3.00 \pm .094 \times 10^{-12}$
0.02	$1.86 \pm .058 \times 10^{-11}$	1000	$1.86 \pm .092 \times 10^{-12}$
0.04	$1.227 \pm .038 \times 10^{-11}$	2000	$1.28 \pm .090 \times 10^{-12}$
0.1	$5.63 \pm .18 \times 10^{-12}$	4000	$9.0 \pm .89 \times 10^{-13}$
0.2	$3.39 \pm .11 \times 10^{-12}$	10000	$6.3 \pm .99 \times 10^{-13}$
0.4	$3.93 \pm .12 \times 10^{-12}$	20000	$4.4 \pm .97 \times 10^{-13}$
1	$5.84 \pm .18 \times 10^{-12}$	40000	$3.4 \pm 1.1 \times 10^{-13}$
2	$7.65 \pm .24 \times 10^{-12}$	100000	$1.7 \pm .87 \times 10^{-13}$
4	$9.82 \pm .31 \times 10^{-12}$		
10	$1.080 \pm .034 \times 10^{-11}$		
20	$1.017 \pm .032 \times 10^{-11}$		
40	$8.28 \pm .26 \times 10^{-12}$		
100	$5.69 \pm .18 \times 10^{-12}$		
200	$4.06 \pm .13 \times 10^{-12}$		

Ch A: 5.0 MHz 3.3 V_{pp}
Averaged Phase

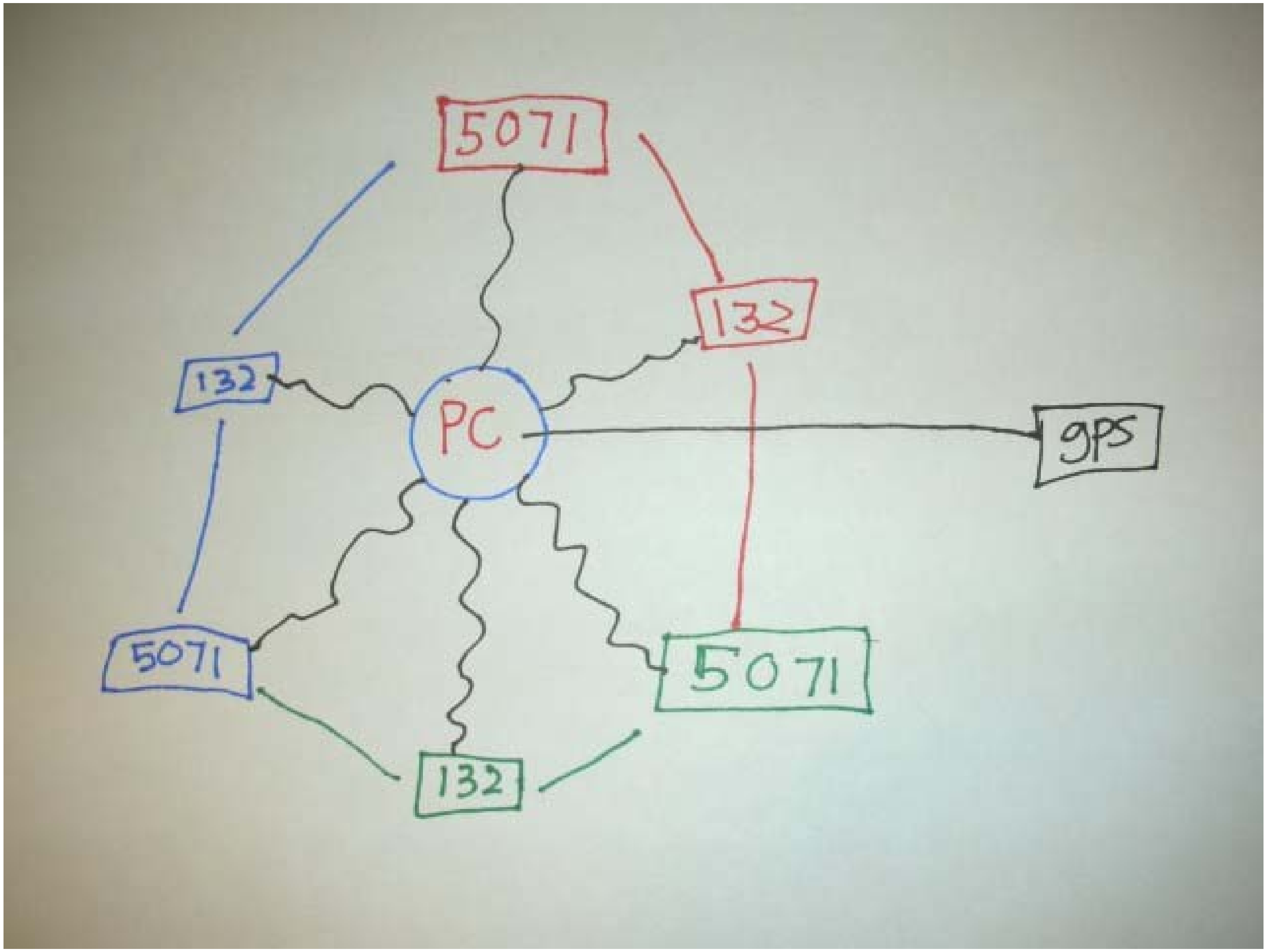
Ch B: 5.0 MHz 2.6 V_{pp}
B/A=Single DDS

c:\tvb\TSCplot\Log22422.gif

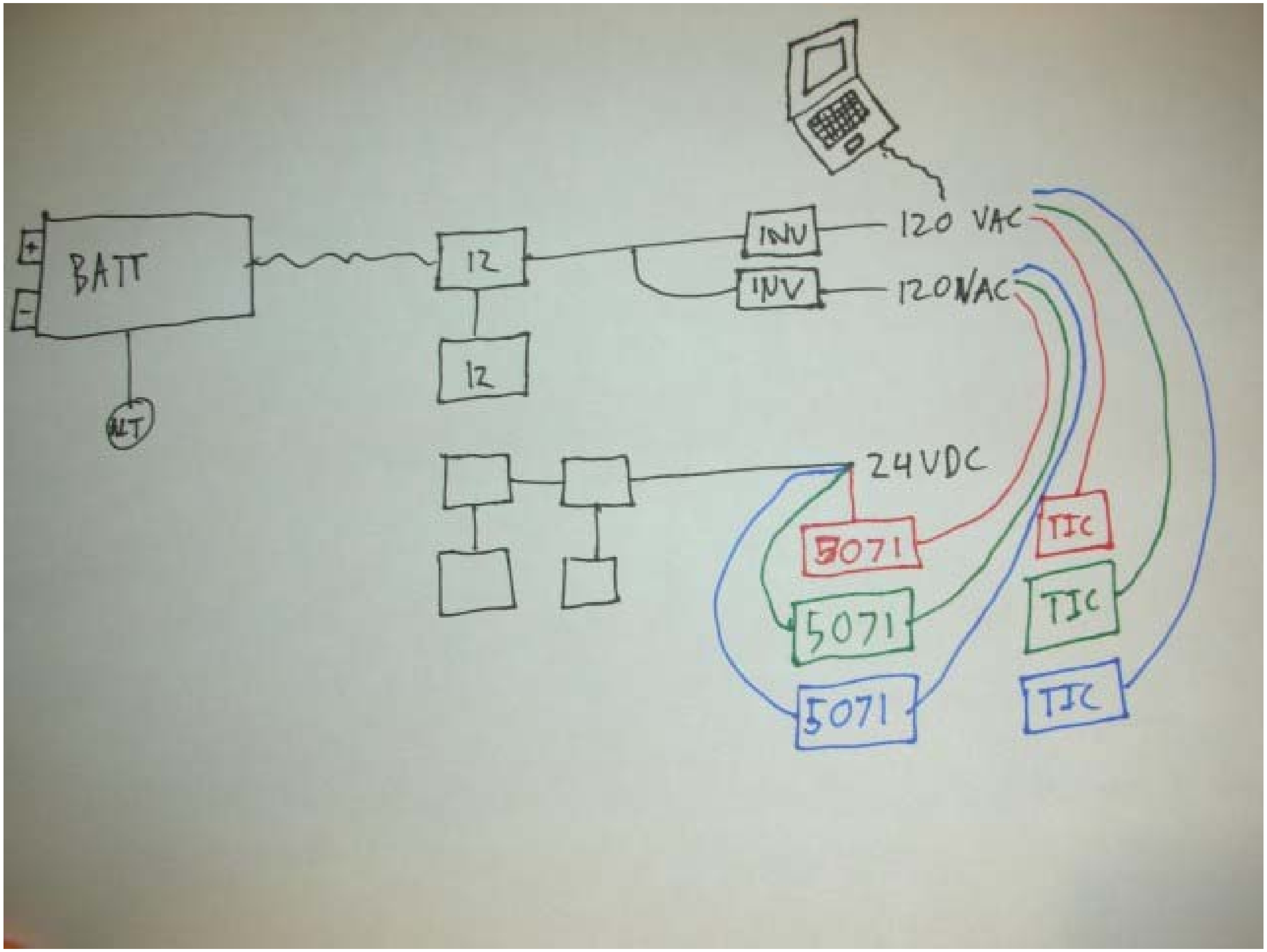
Map – home clocks



Map – mobile clocks



Map – mobile power



Map – Paradise, Mt Rainier

- ~

Chapter 7

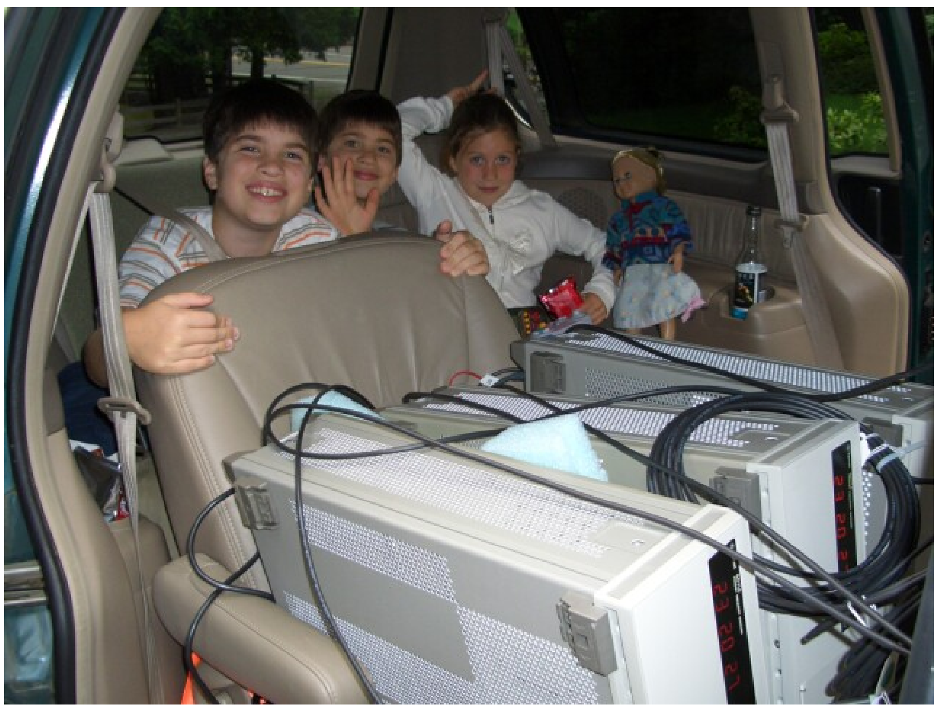
- Photos of the trip

The GREAT Trip, day 1

- Carrying clock downstairs. Limited time; car is a mess, but it works.

The GREAT Trip, day 1

- Kids in the rear, clocks in the middle, and instrumentation in the front.



The GREAT Trip, day 1

- Dad making final clock BNC connections to TIC; Mom says goodbye.

The GREAT Trip, day 1

- Detail of TIC's and laptop in front seat and clocks in middle seat. 23:33:48 UTC

The GREAT Trip, day 1

- Kids drink stop costs me \$8 and 125 ps ($\frac{1}{4}$ hour x 500ps/h).

The GREAT Trip, day 1

- Final gas stop and evening arrival in Rainier National Park.

The GREAT Trip, day 2

- Paradise Inn is at 5400' elevation. Large parking lot to hide in.

The GREAT Trip, day 2

- Classic old Northwest inn; you should visit sometime.

The GREAT Trip, day 2

- Wonderful hiking trails and climbing.


The GREAT Trip, day 2

- Good, the car is still there. Hike to Glacier Vista (6300')

The GREAT Trip, day 2

- Oh no. The sun is really strong and the A/C isn't working as well as I hoped.

The GREAT Trip, day 2

- Avoid a ticket and move the car again. Ouch, running low in fuel. Now what.

The GREAT Trip, day 2

- Kids are fine. Trip is long. Looking for GPS location for next time. What! Valet shuts car off for an hour.

The GREAT Trip, day 2

- Air is thin; little sleep; ponder Time; spend an hour; why are we doing this; Harrison; GPS. That's 500 ps/beer.

The GREAT Trip, day 3

- Got gas at 6 AM. Used 15.78 gal in 34 h = 0.46 gph; ~2h/gal, so about 1 ns/gal. Cost me $51 and 745 ps.

The GREAT Trip, day 3

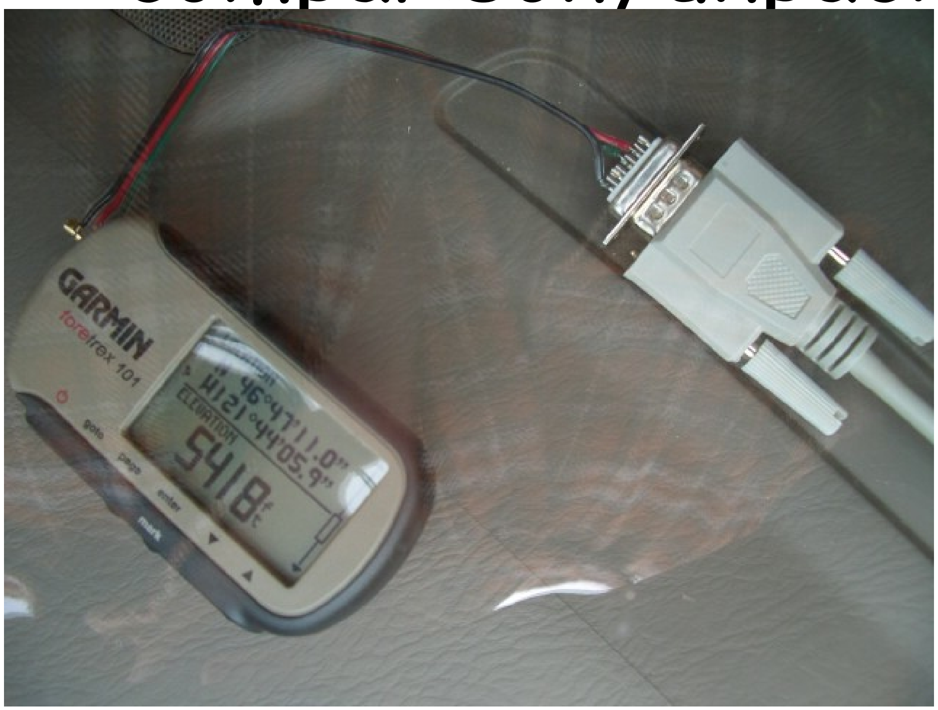
- More hiking, exploring, playing. It's a fun place for a while.

The GREAT Trip, day 3

- 42 hours is up; time to leave. We're all tired. Can this really work? Go home.

The GREAT Trip, day 3

- Carry clocks & TIC's back inside, reconnect same cables, resume phase comparison, unpack car. Sleep.



Chapter 8

- Analysis of GPS data

GPS Log

- Serial NMEA data stream (0.5 Hz)
- $GPGGA (time/lat/lon/alt)
- $GPRMC (time/date/lat/lon/speed)

53632.503380 $GPRMC,120454,A,4644.1107,N,12151.8606,W,26.8,272.6,190905,17.8,E,A*03

53632.503380 $GPGGA,120454,4644.1107,N,12151.8606,W,1,03,2.2,727.7,M,-18.8,M,,*77

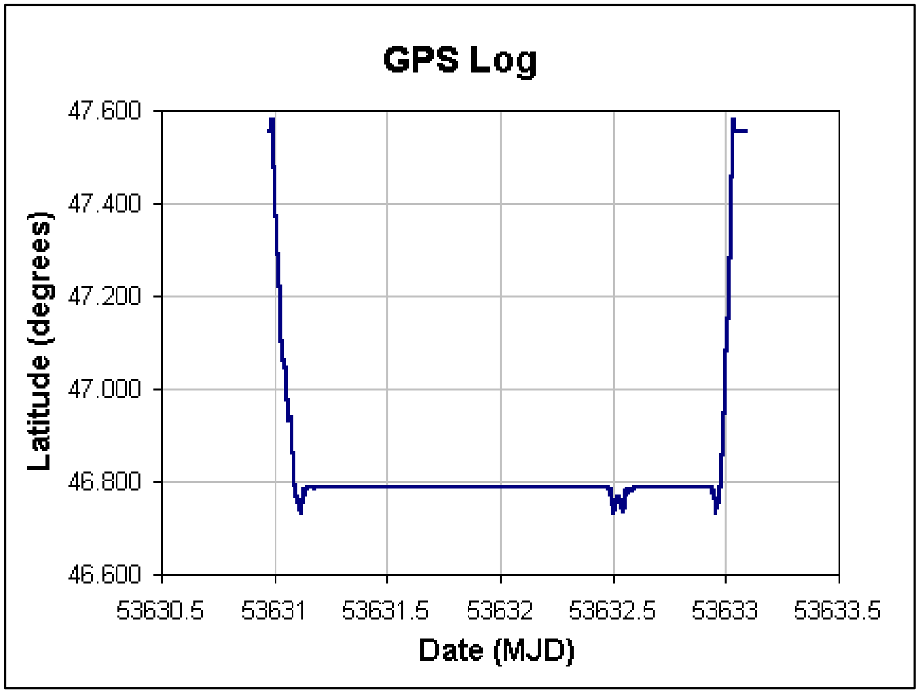
- Software tool calculates time dilation

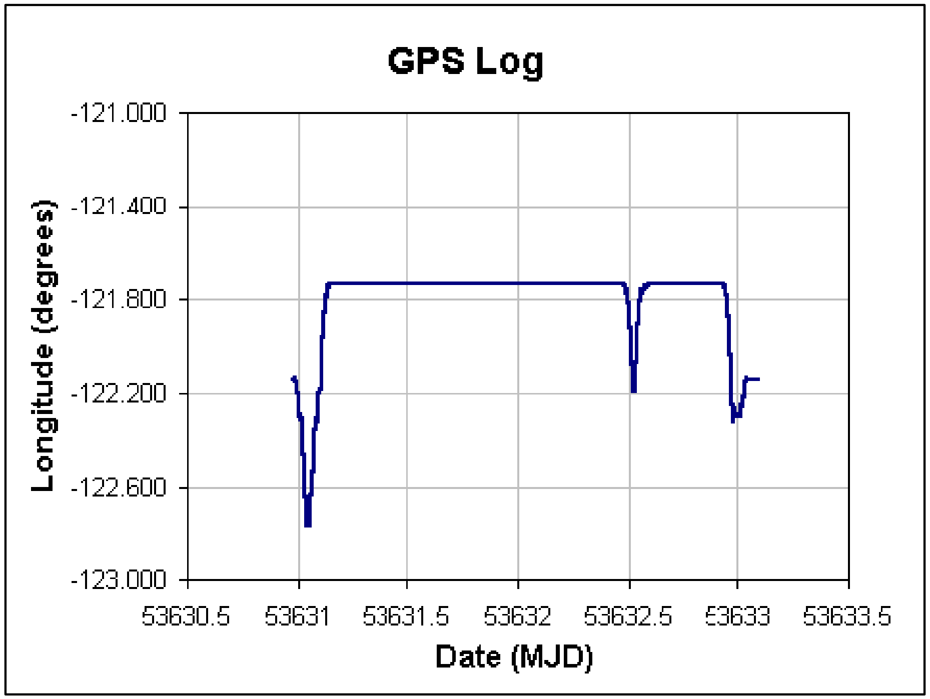
53632.503380 46.7352,-121.8640,728,14,0 17.325260 -0.027115 -0.076591 17.222

- Generated plots

Plots from GPS Log

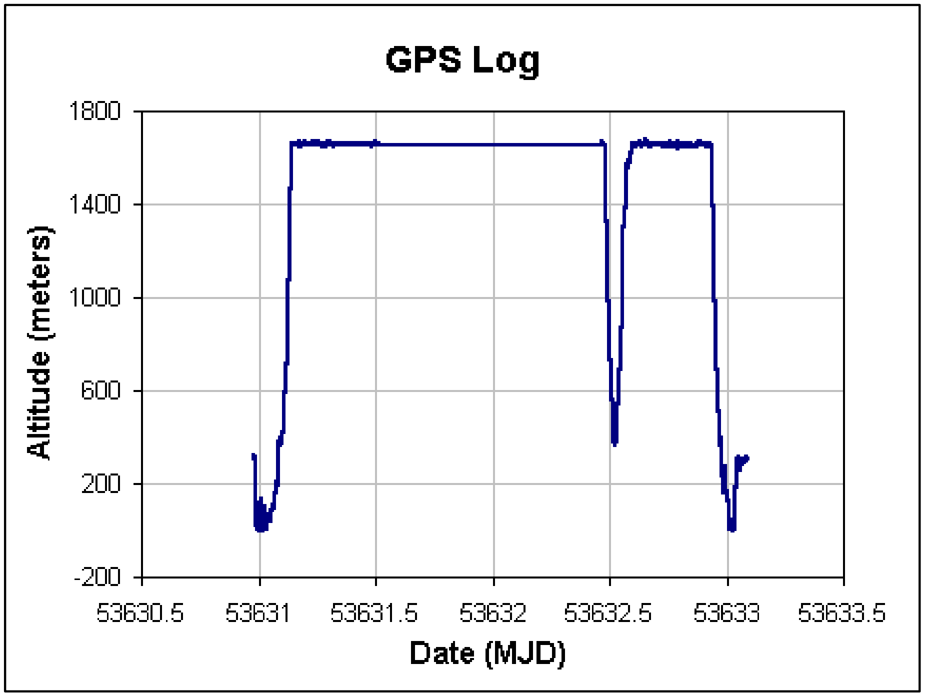
- Latitude, Longitude

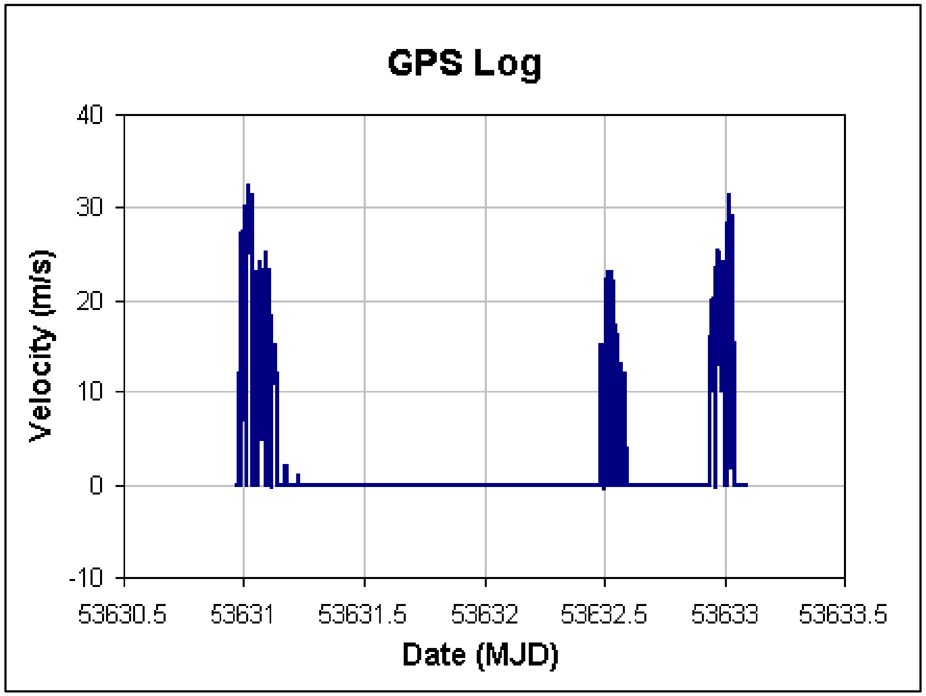




Plots from GPS Log

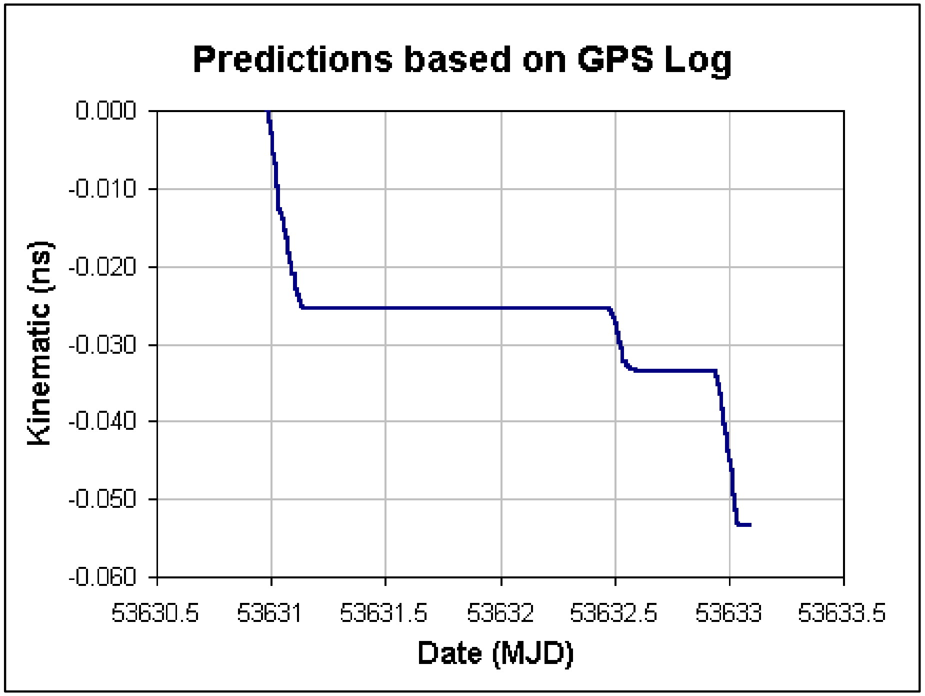
- Altitude, Velocity

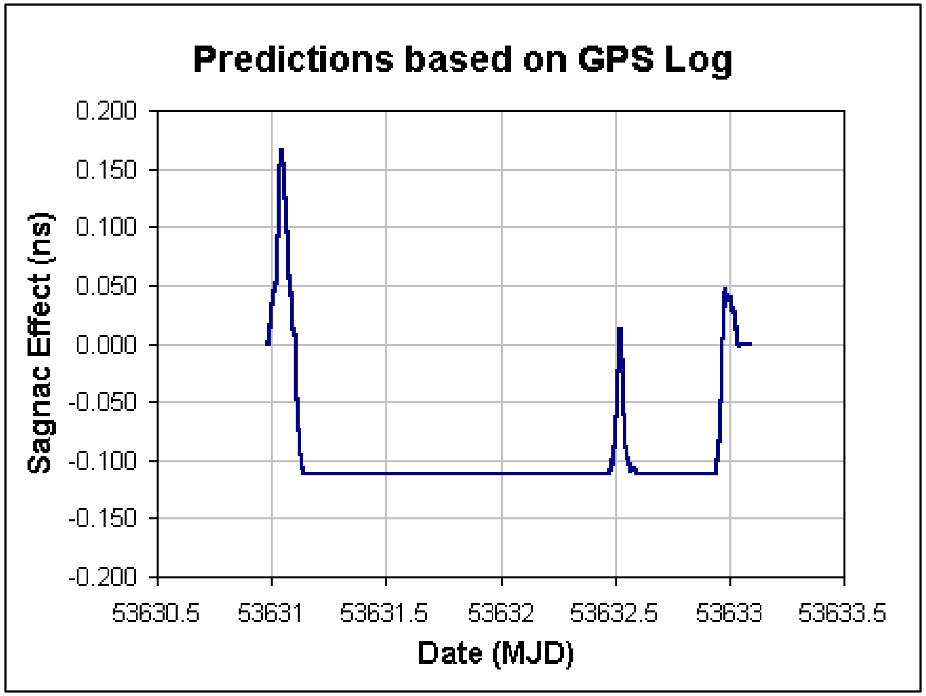




Predictions from GPS Log

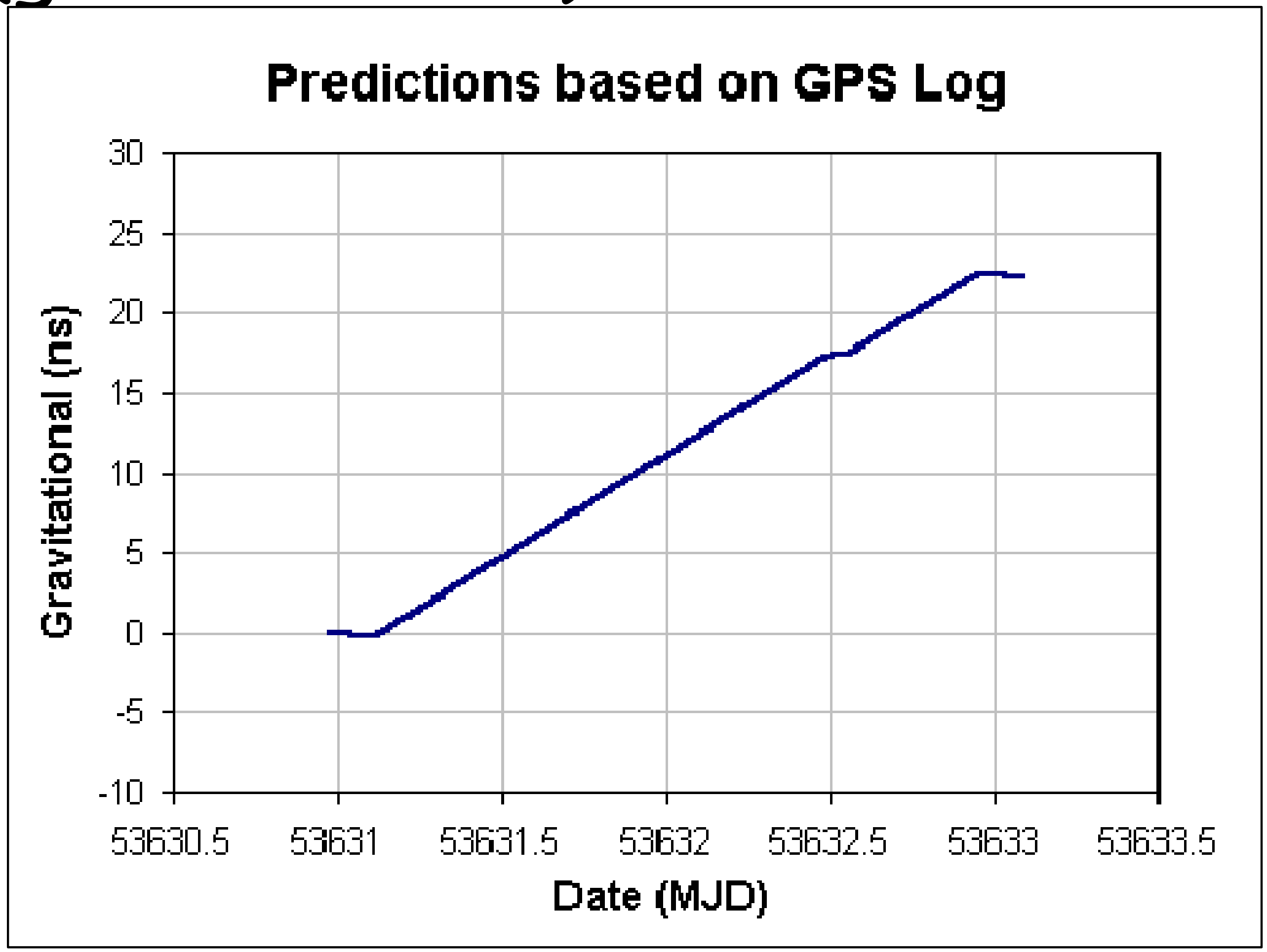
- SR (velocity): 50 ps
- Sagnac effect: ±150 ps (net 1 ps)





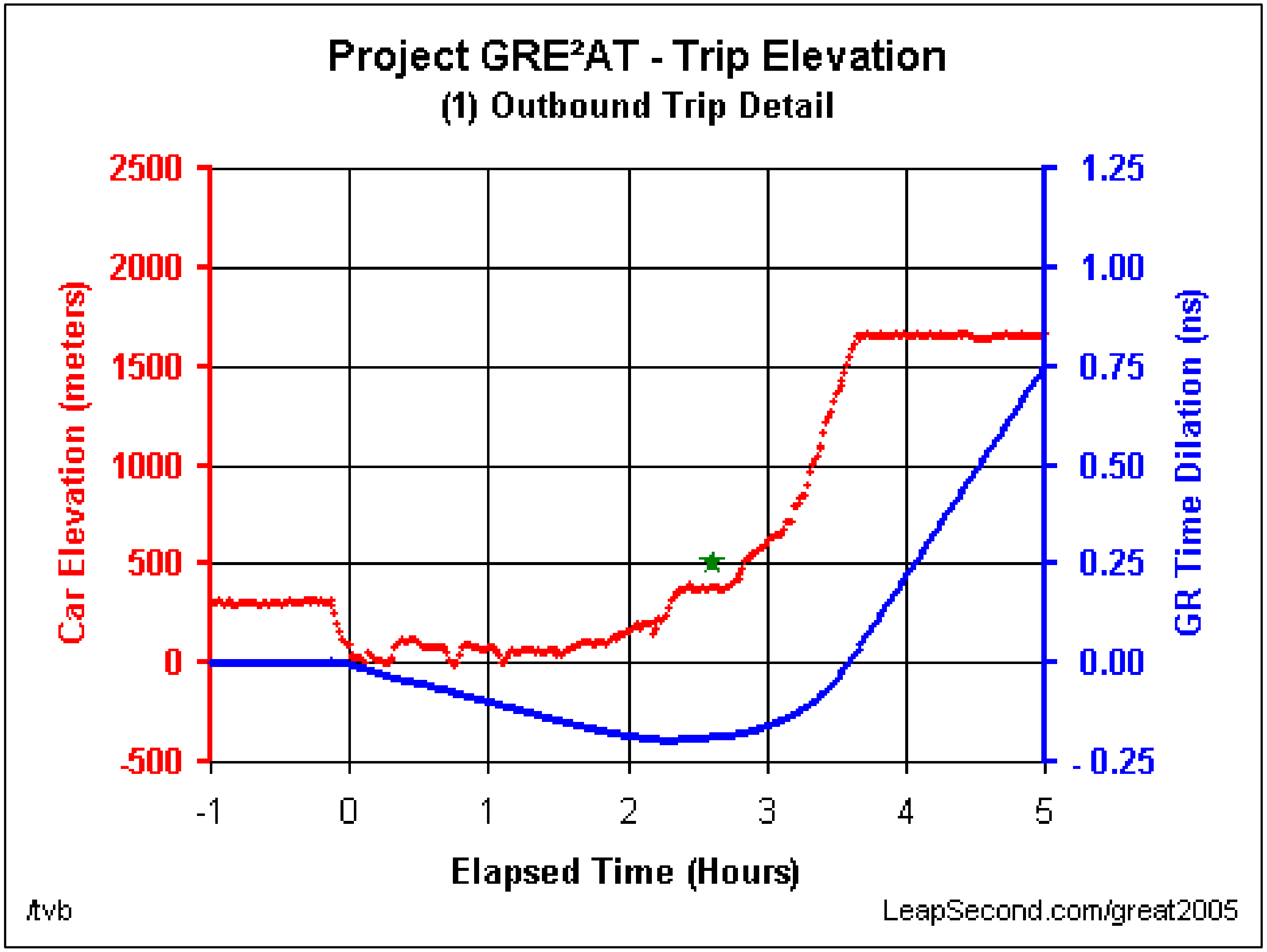
Predictions from GPS Log

- GR (gravitational): 22.37 ns



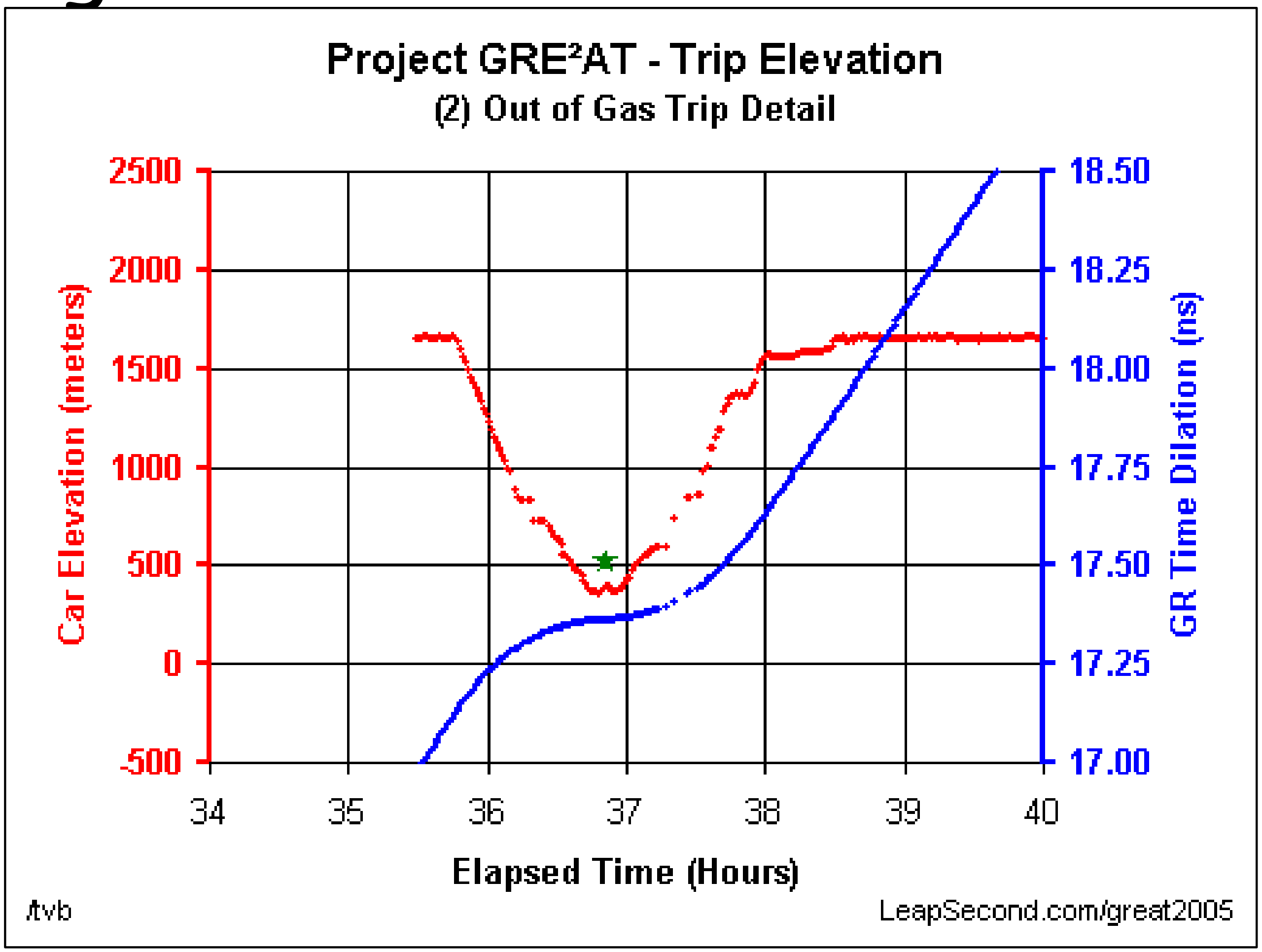
Composite plot - beginning

- Trip start



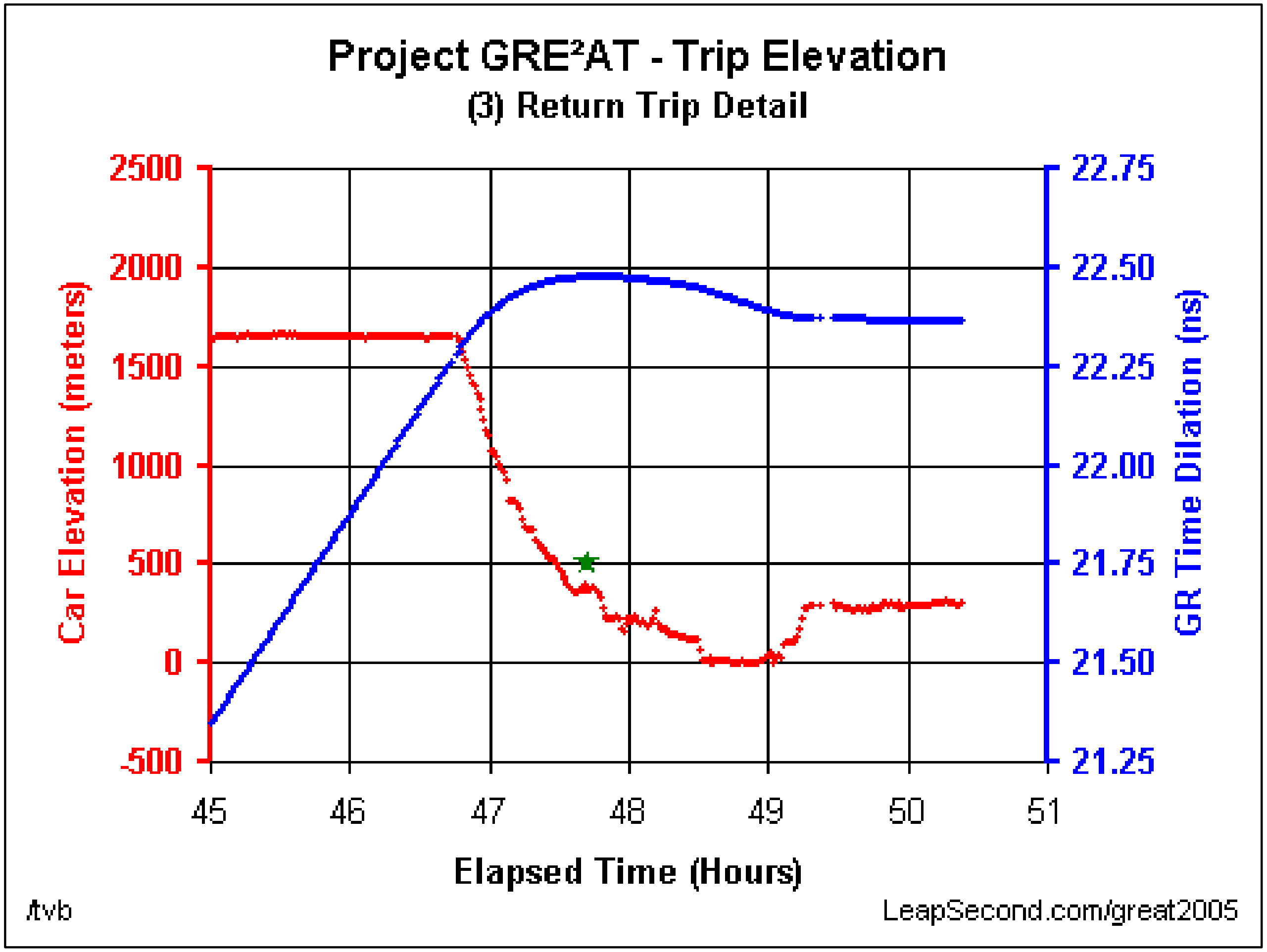
Composite plot - middle

- More gas

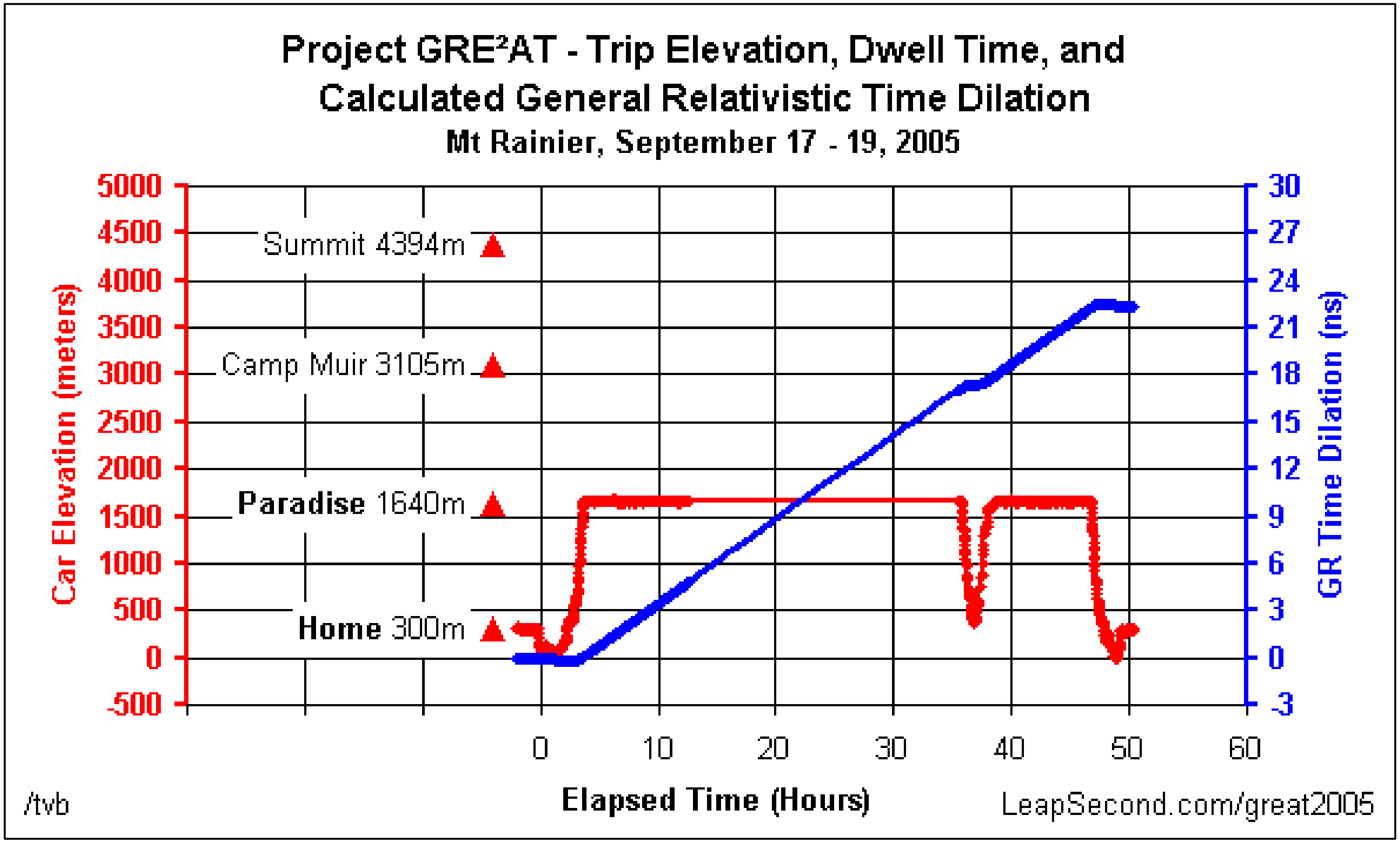


Composite plot - end

- Trip end



Composite plot (net 22.32 ns)

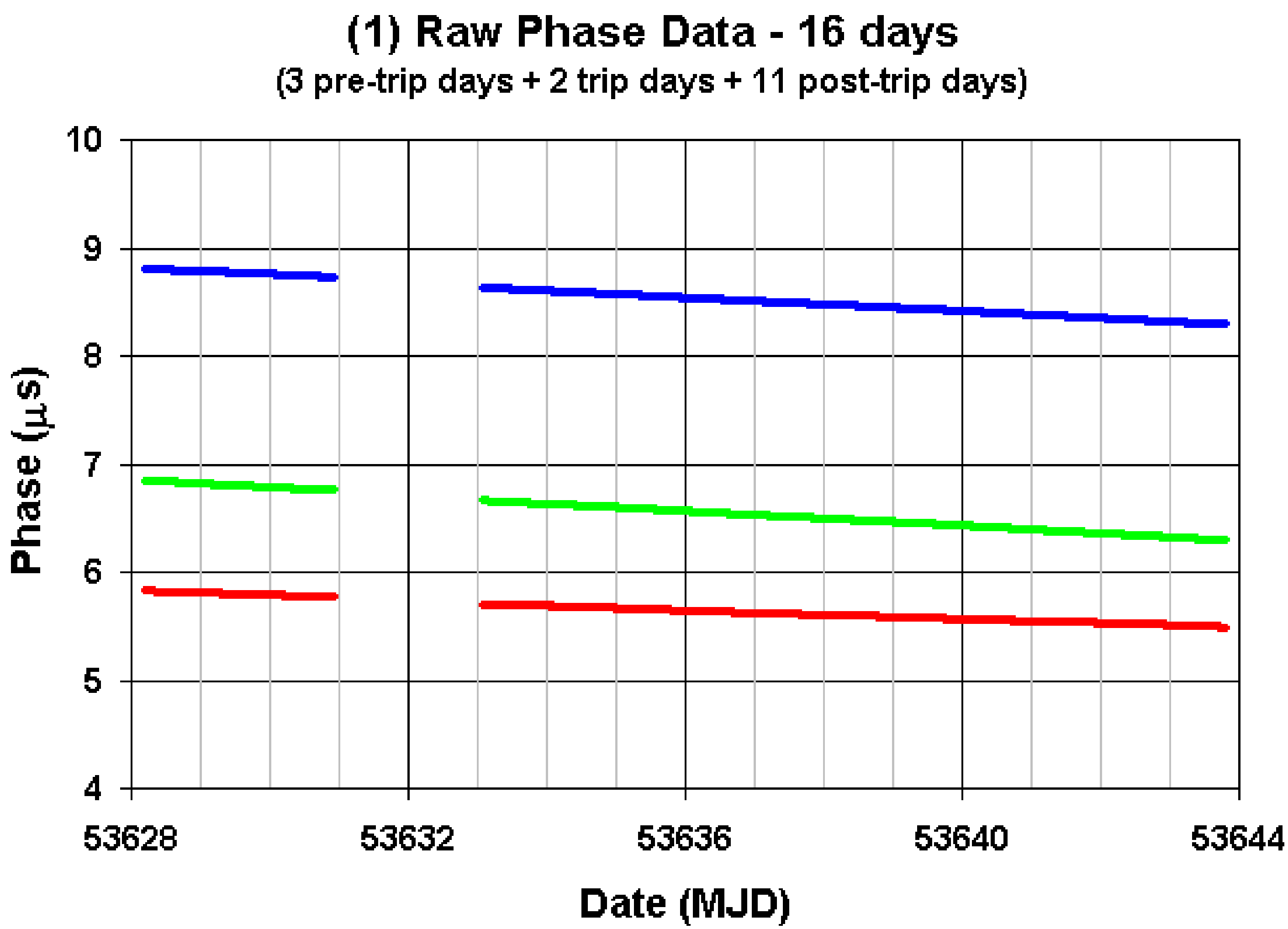


Chapter 9

- Analysis of clock data

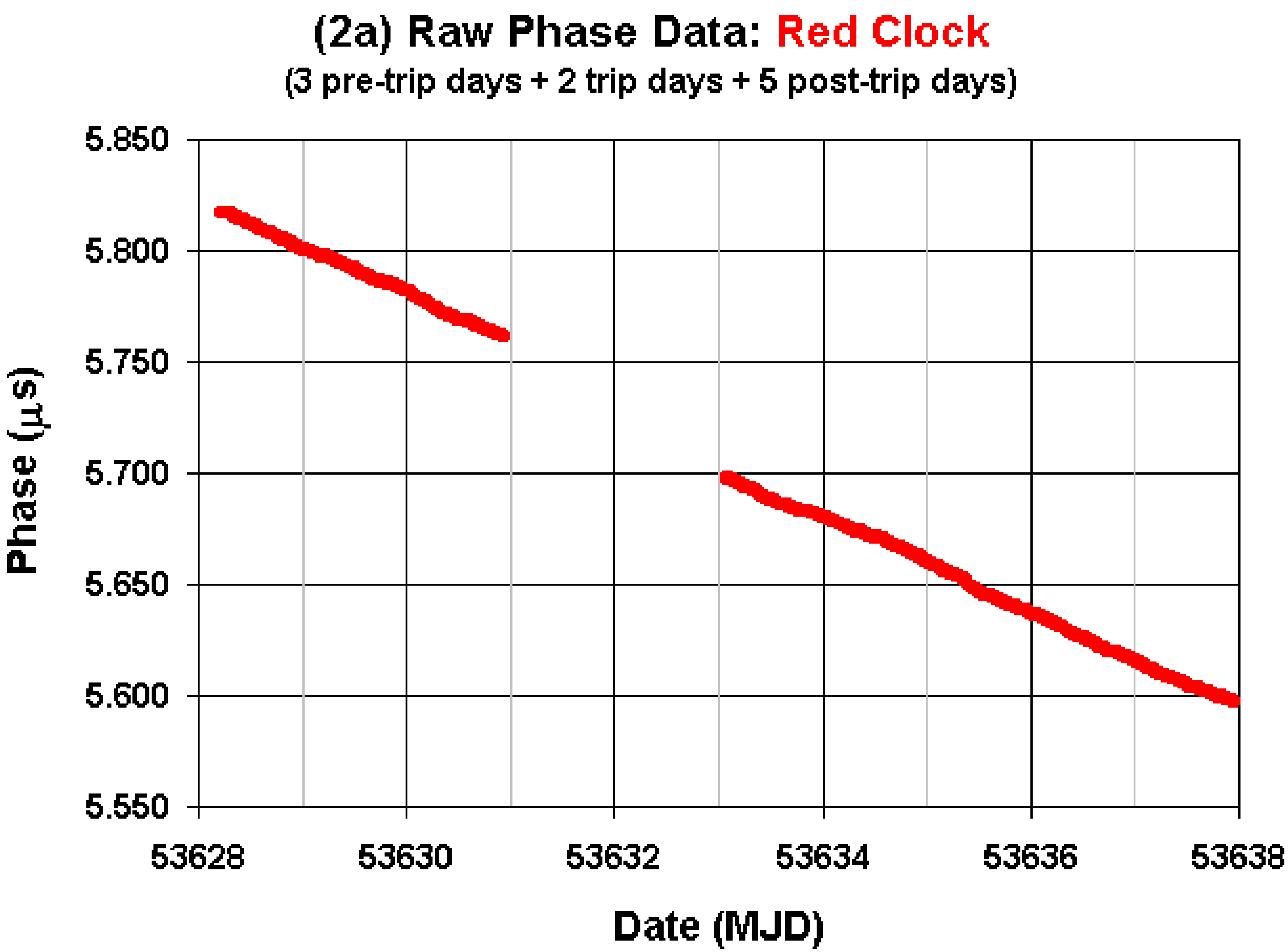
Worked example

- Raw data
- 3 clocks



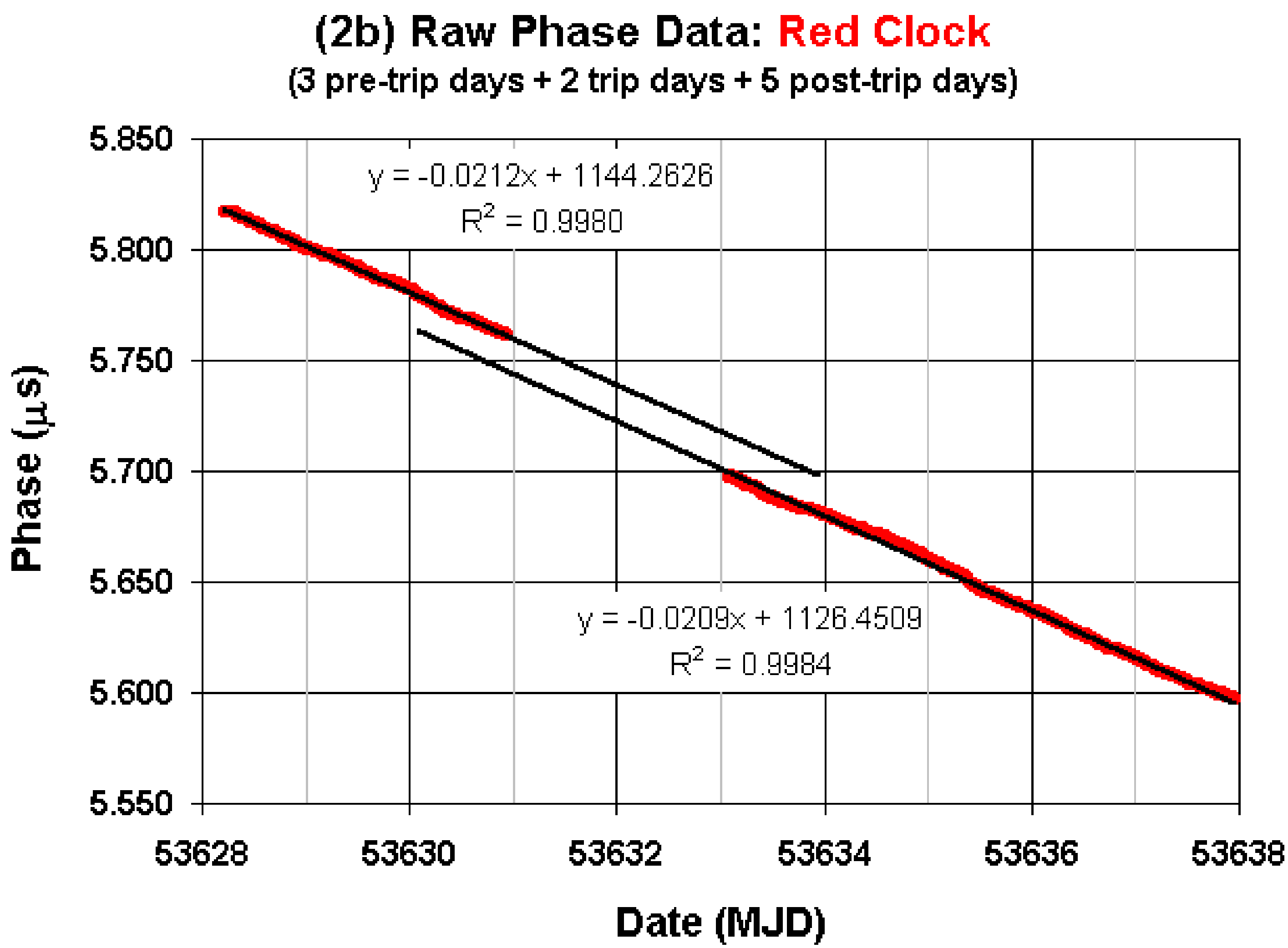
Worked example

- Red only
- 5.85 μs
  5.55 μs
- -220 ns
- 10 days



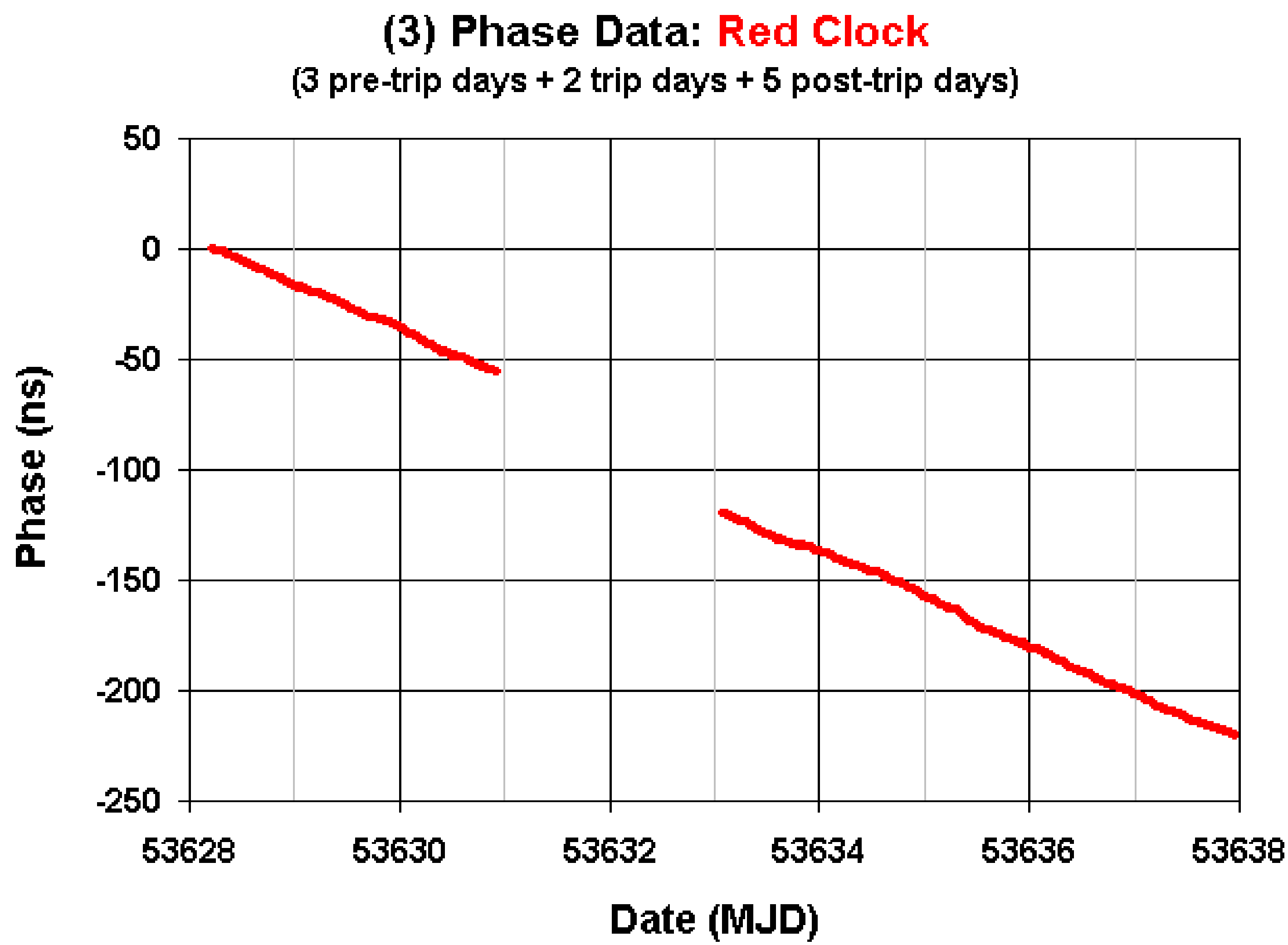
Worked example

- Slope (freq)
- Pre/post
- -21.2
  -20.9
  ns/d
- 2.454
  2.419
  $\times 10^{-13}$



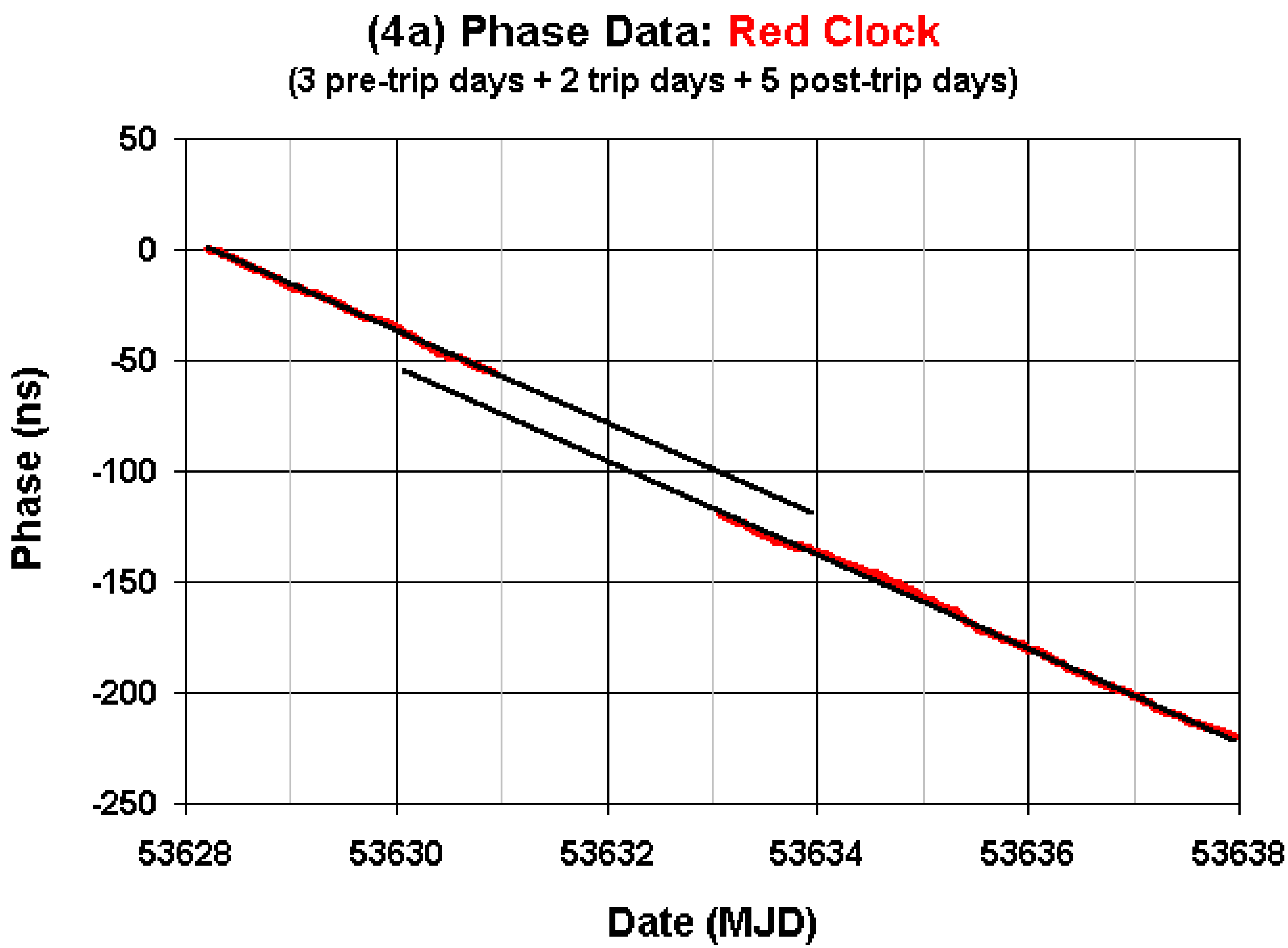
Worked example

- Remove fixed ~5.8μs offset



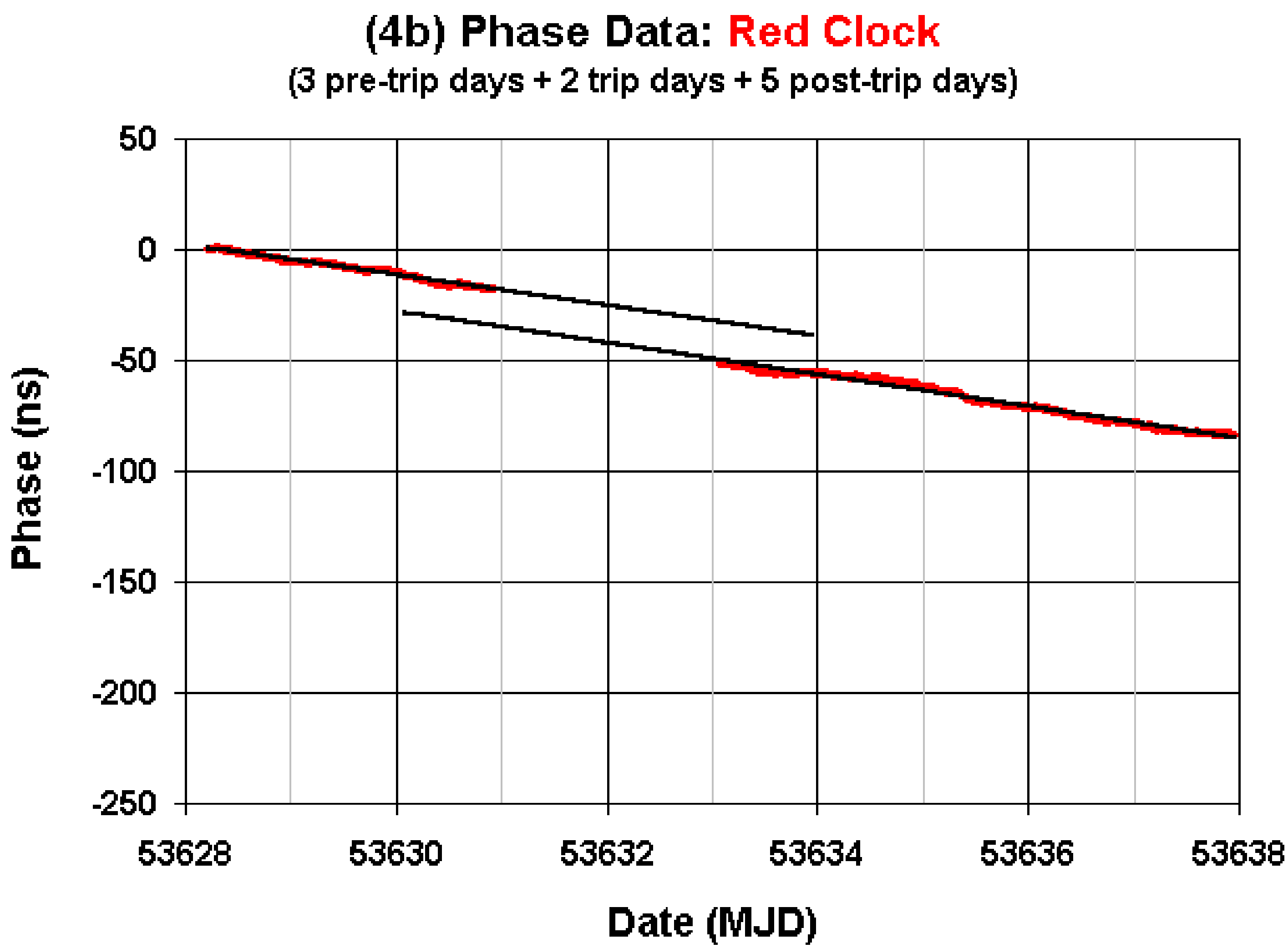
Worked example

- Line fit
- 0×10^{-13}
- Phase offset
- No freq diff



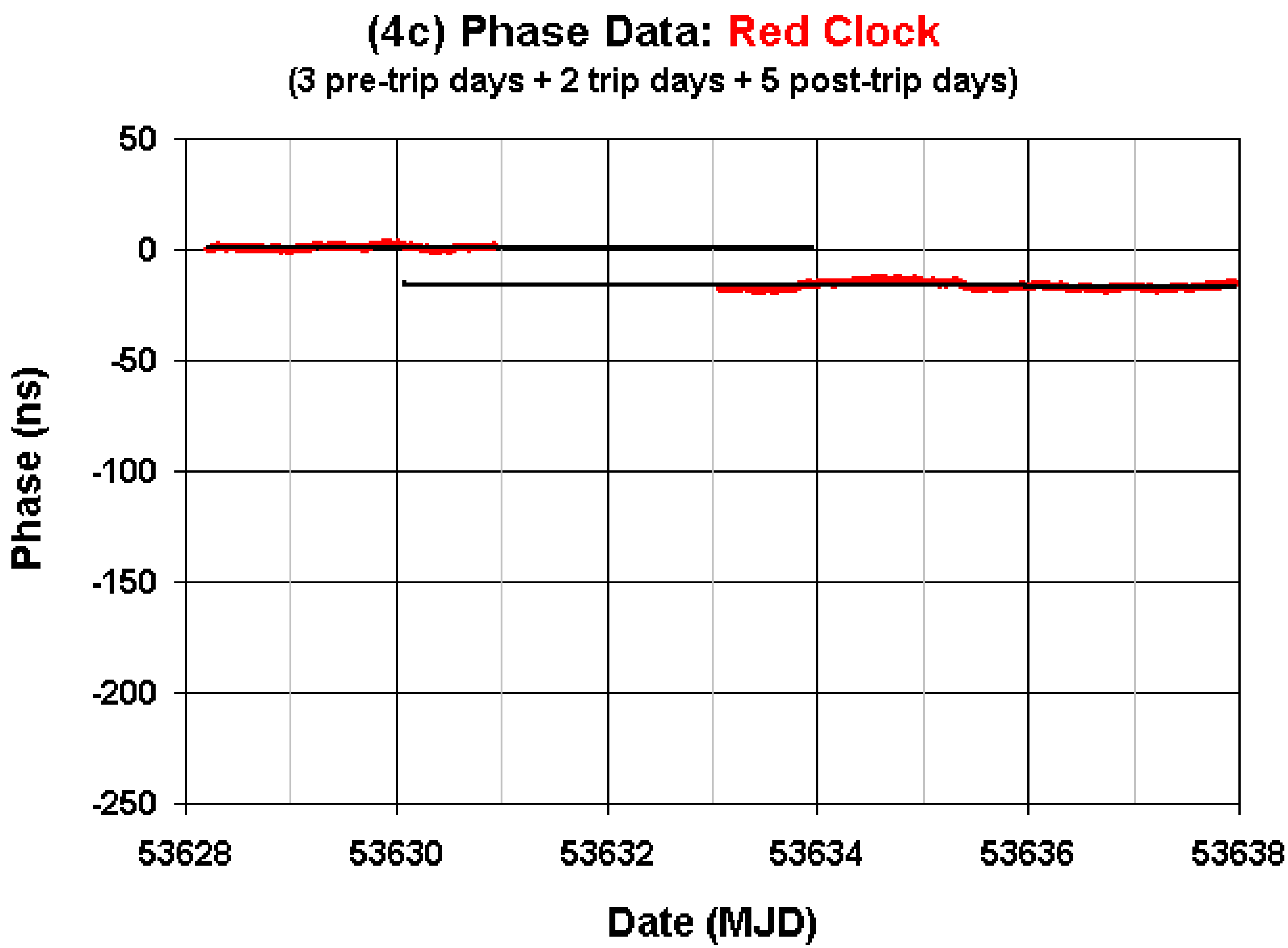
Worked example

- More fit
- 1.62×10^{-13}



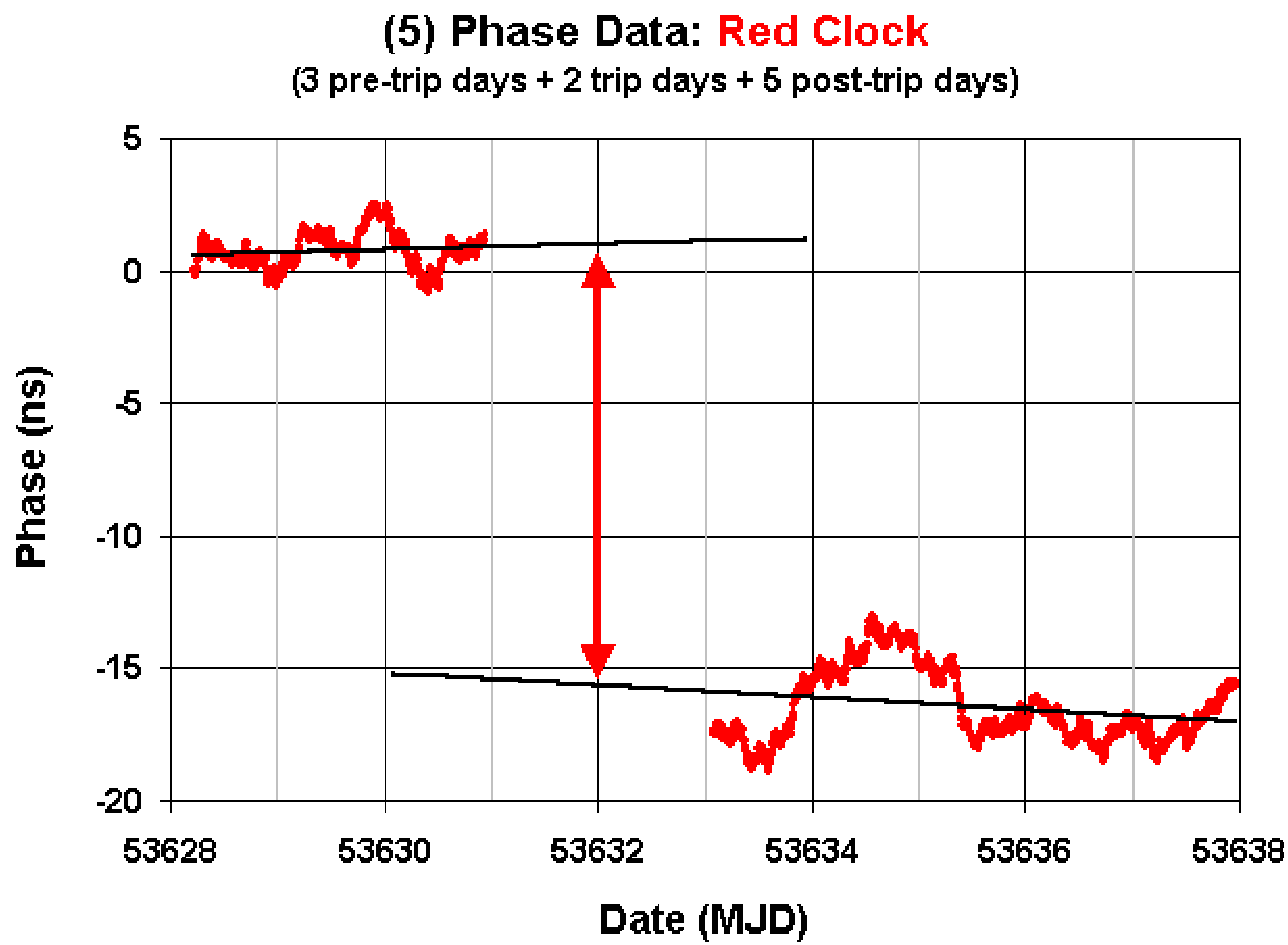
Worked example

- Best fit
- 2.43 $\times 10^{-13}$



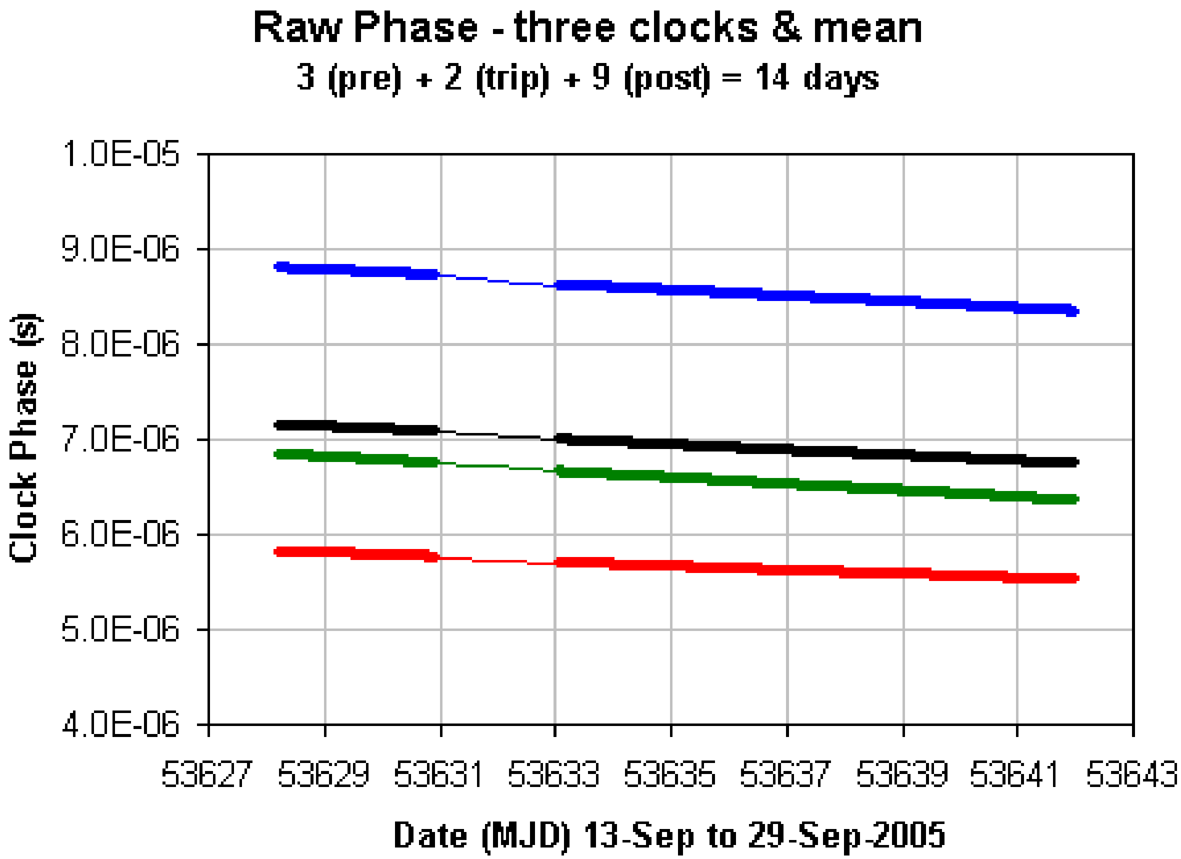
Worked example

- Expand scale
- ~1 ns/d match
- Huge time jump!



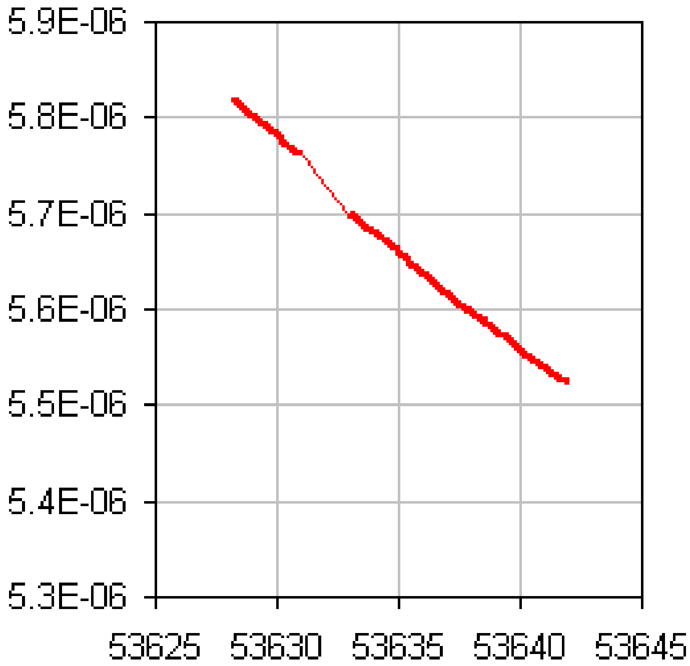
Results

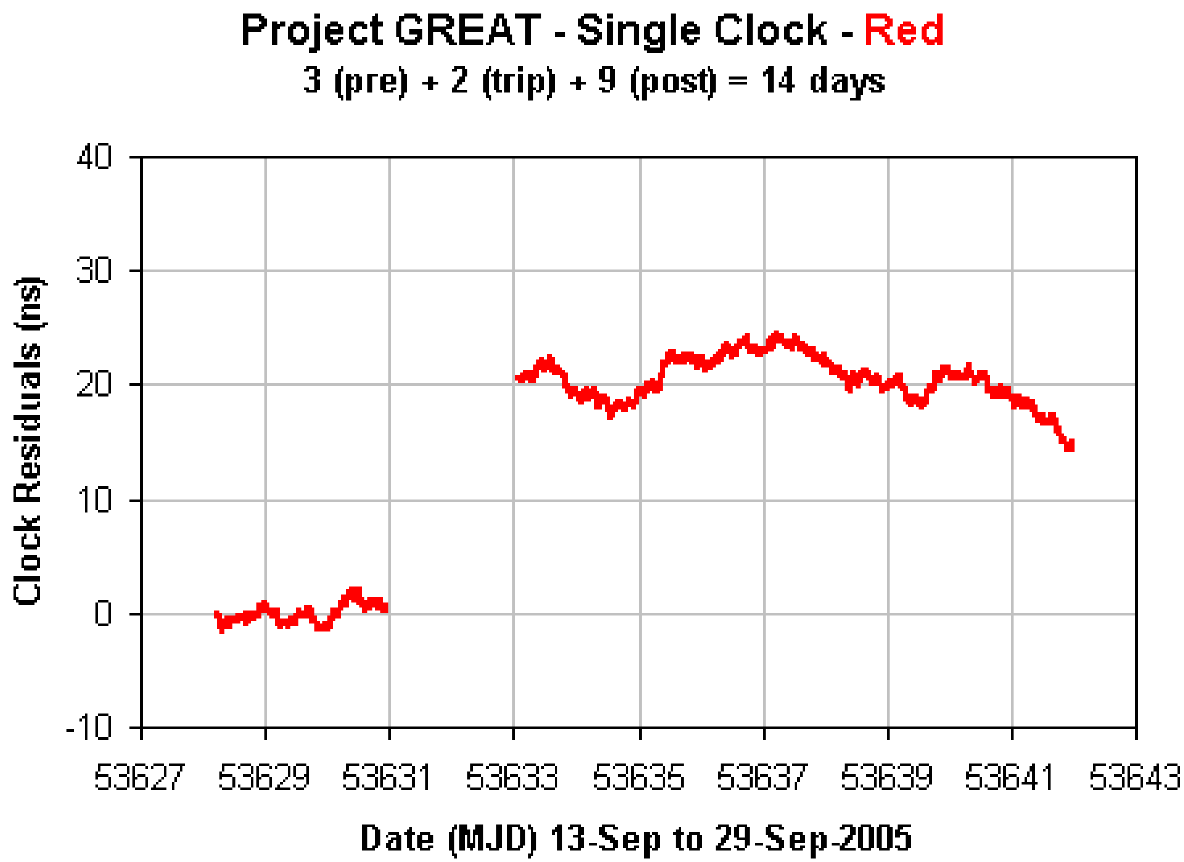
- Raw clock phase, with mean



Results

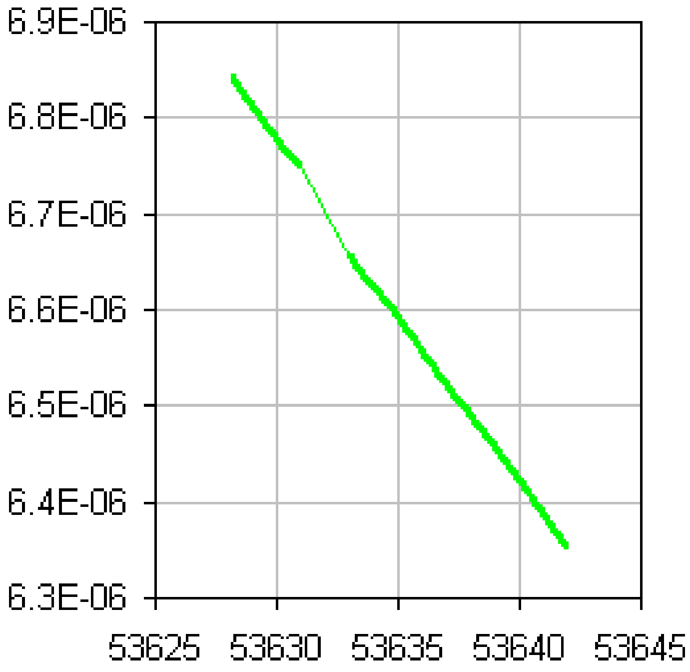
- Red
- 20.3 ns

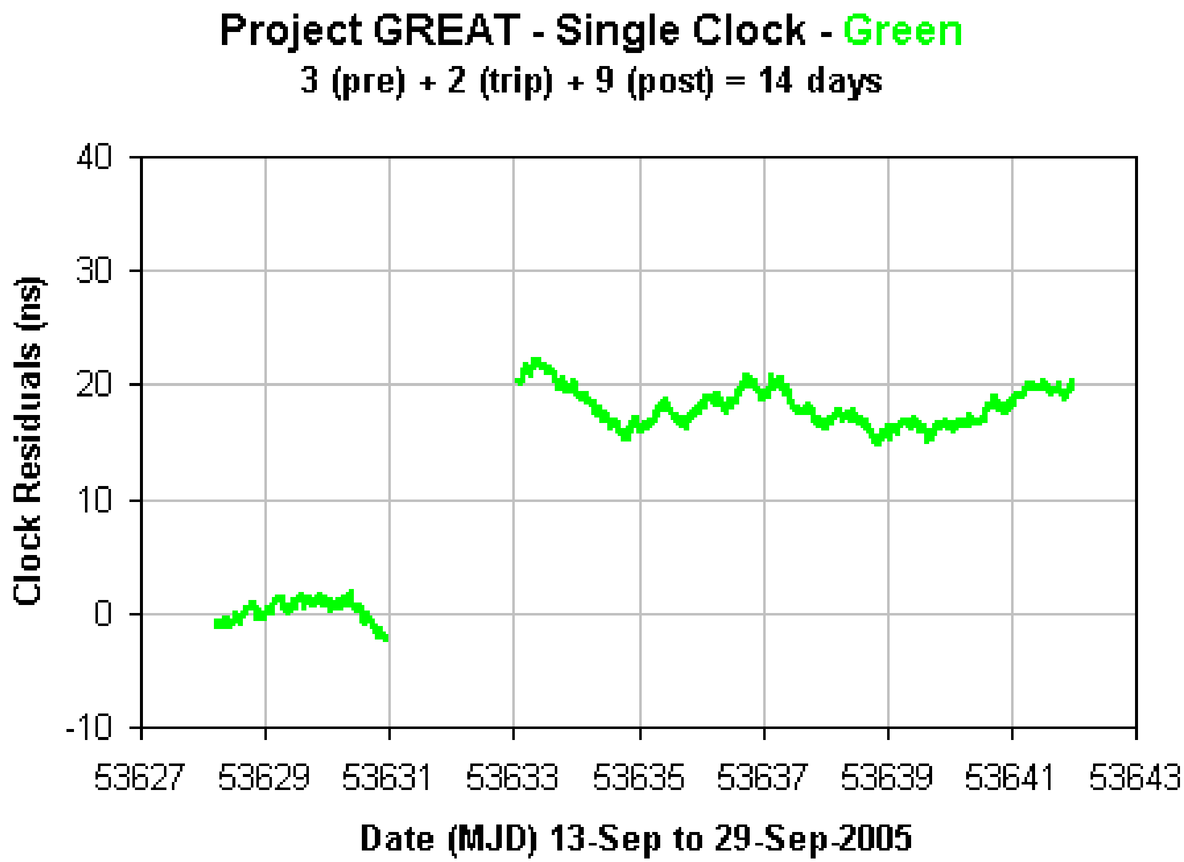




Results

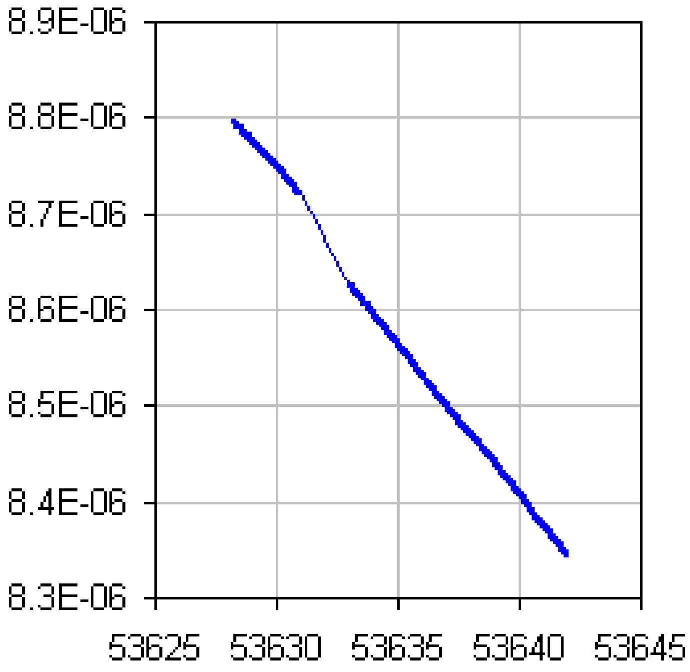
- Green
- 17.5 ns

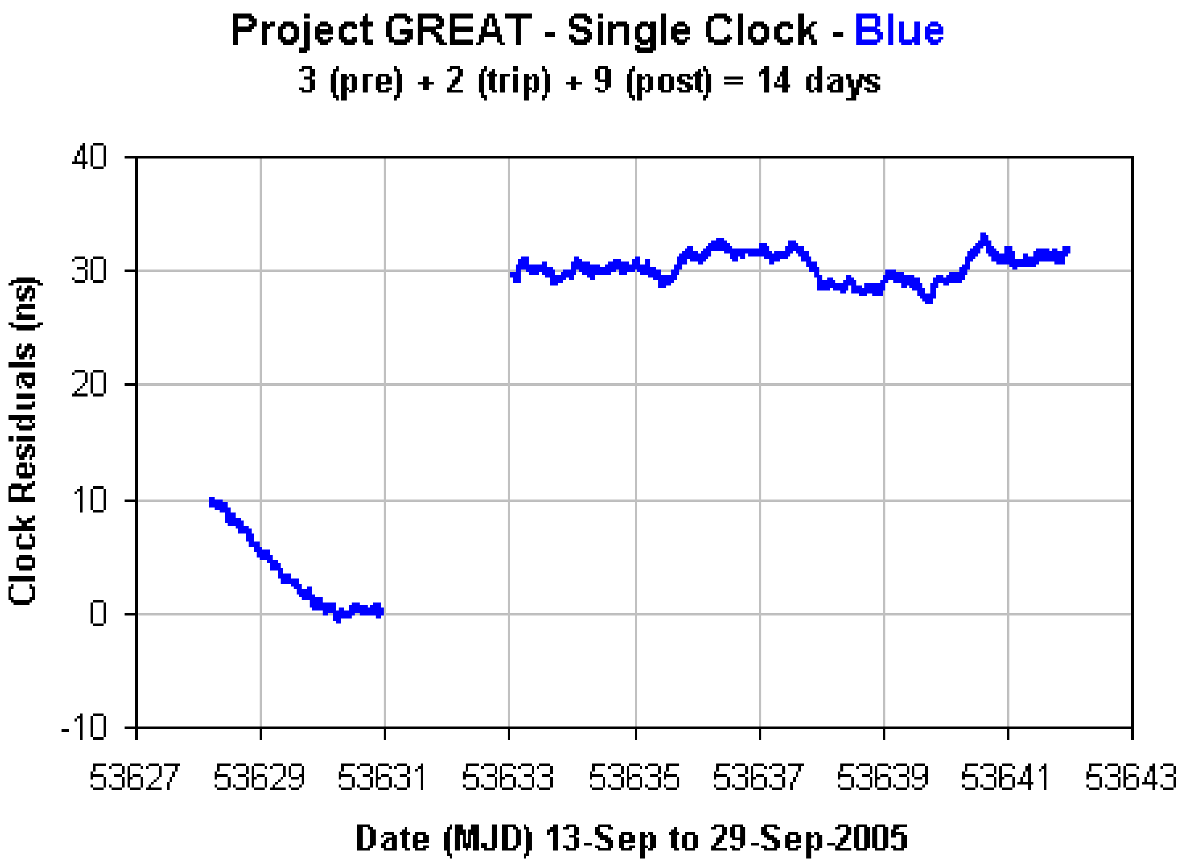




Results

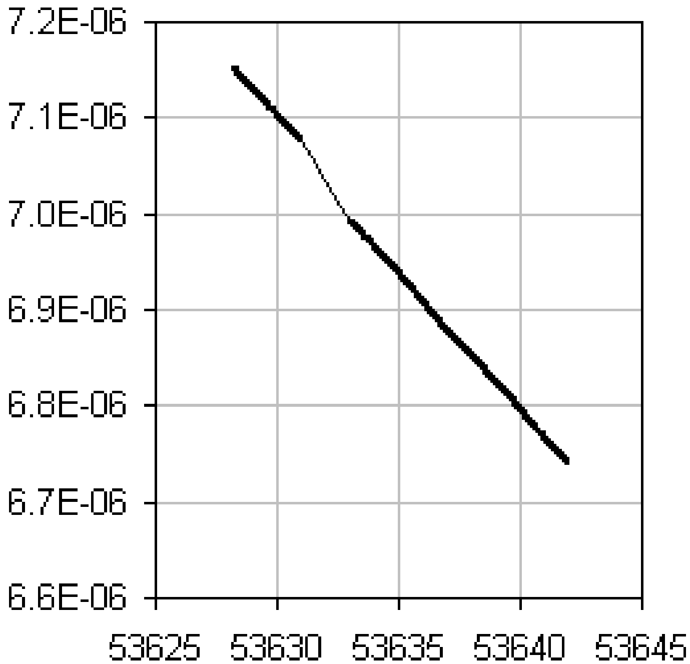
- Blue
- 26.3 ns
- 29.7 ns

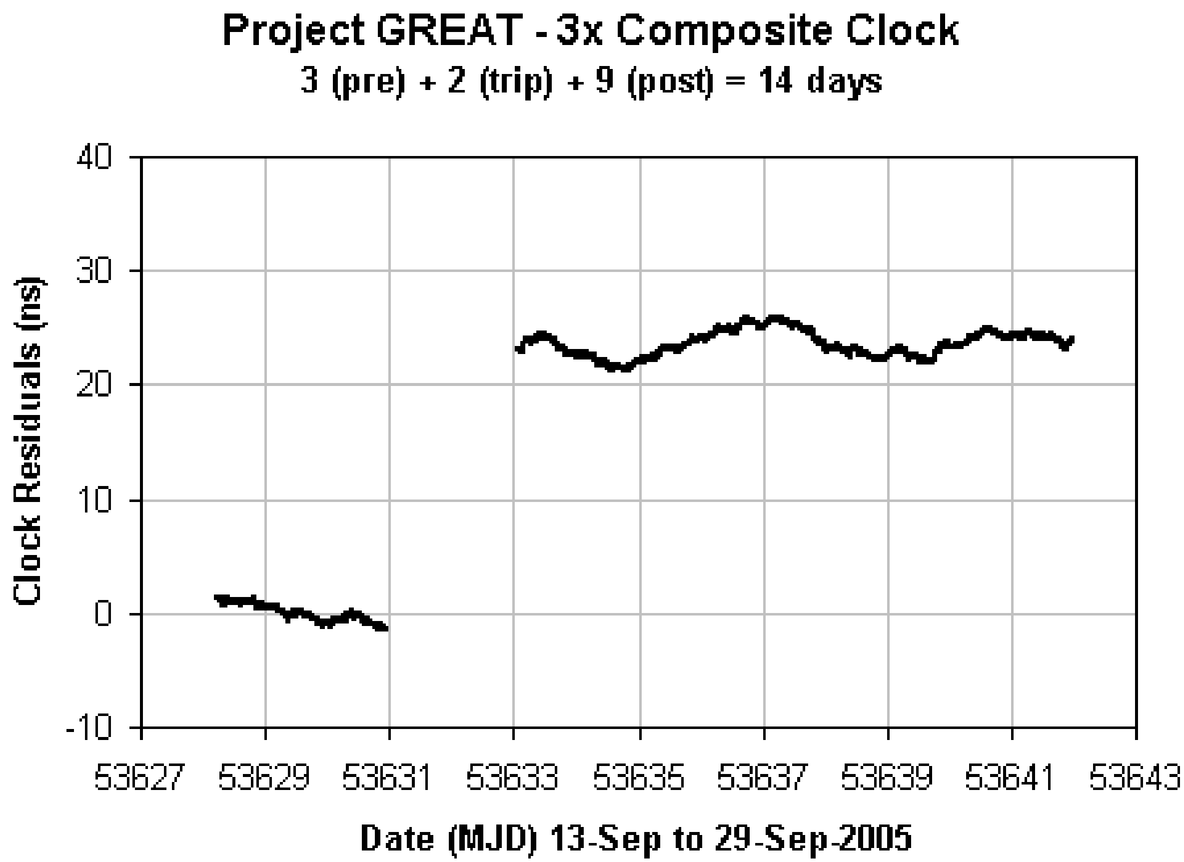




Results

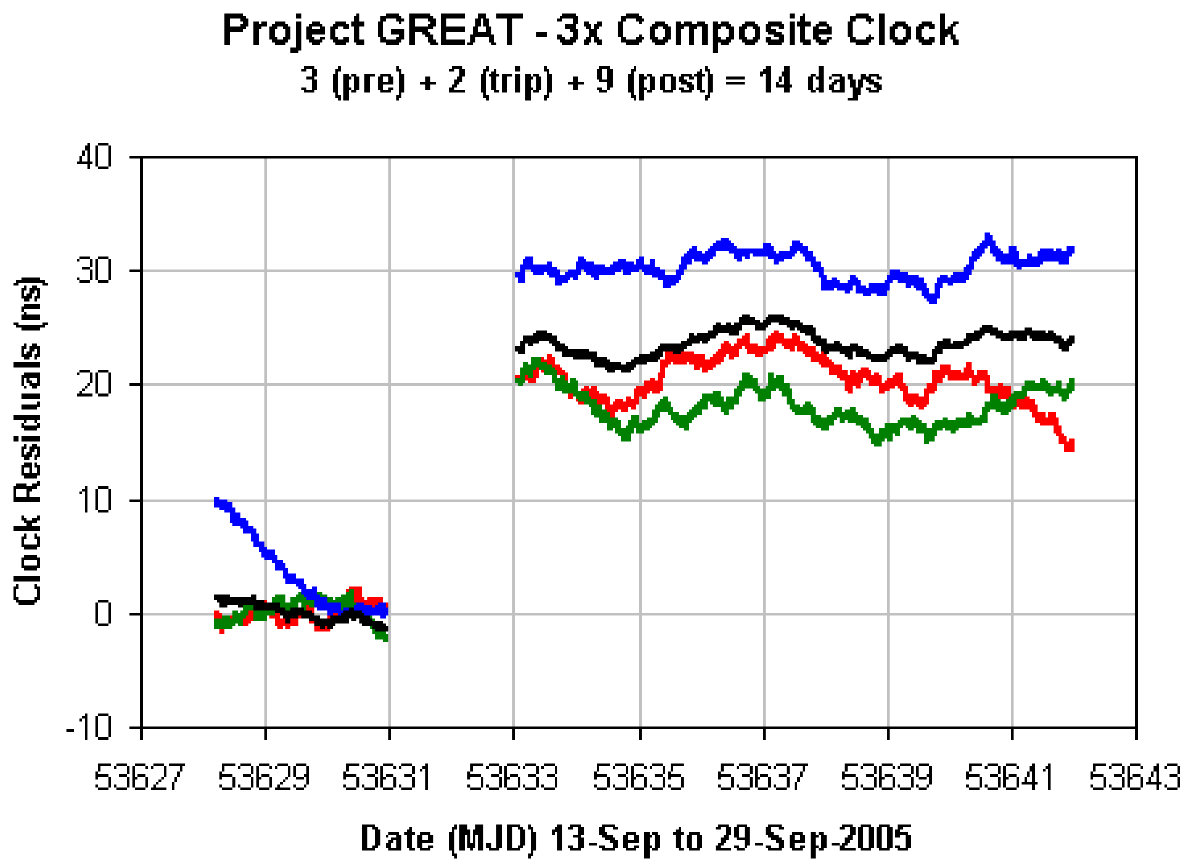
- Mean
- 23.2 ns





Results

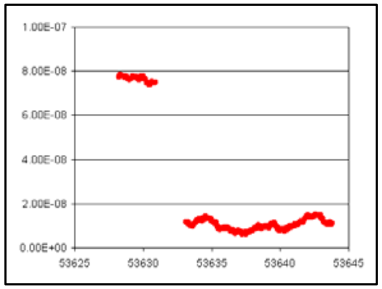
- Composite
- ±4 ns(?)



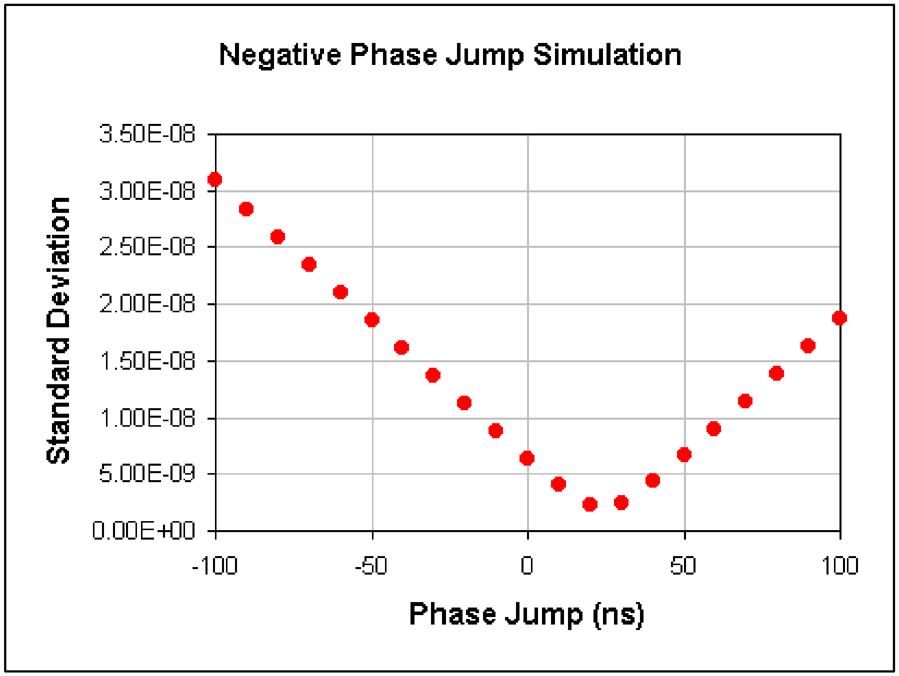
Analysis Methods

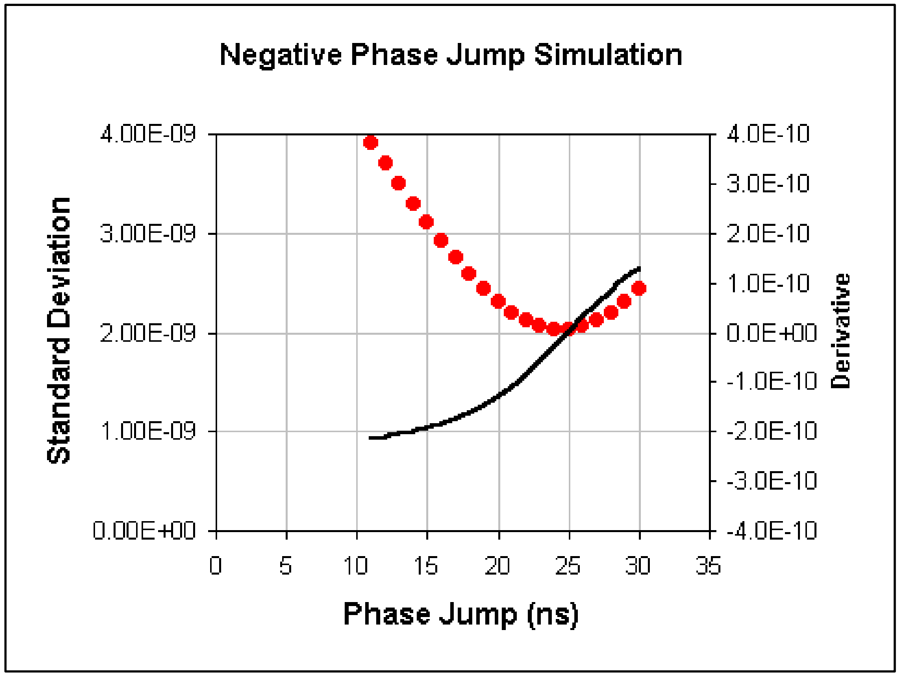
- Frequency before extrapolated forward
- Frequency after extrapolated backward
- Both before and after; mean frequency
- Continuous or hourly/daily averages?
- One day; or ±3 days; -n +m days?
- Mean phase or last/first phase?
- Calculated removal of phase jump until optimal least squares fit?

Phase Jump Simulation



- Artificially remove time dilation
- Stddev improves to a point



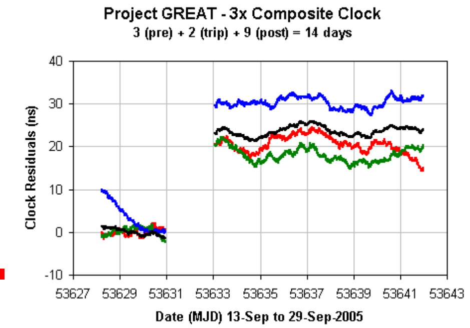


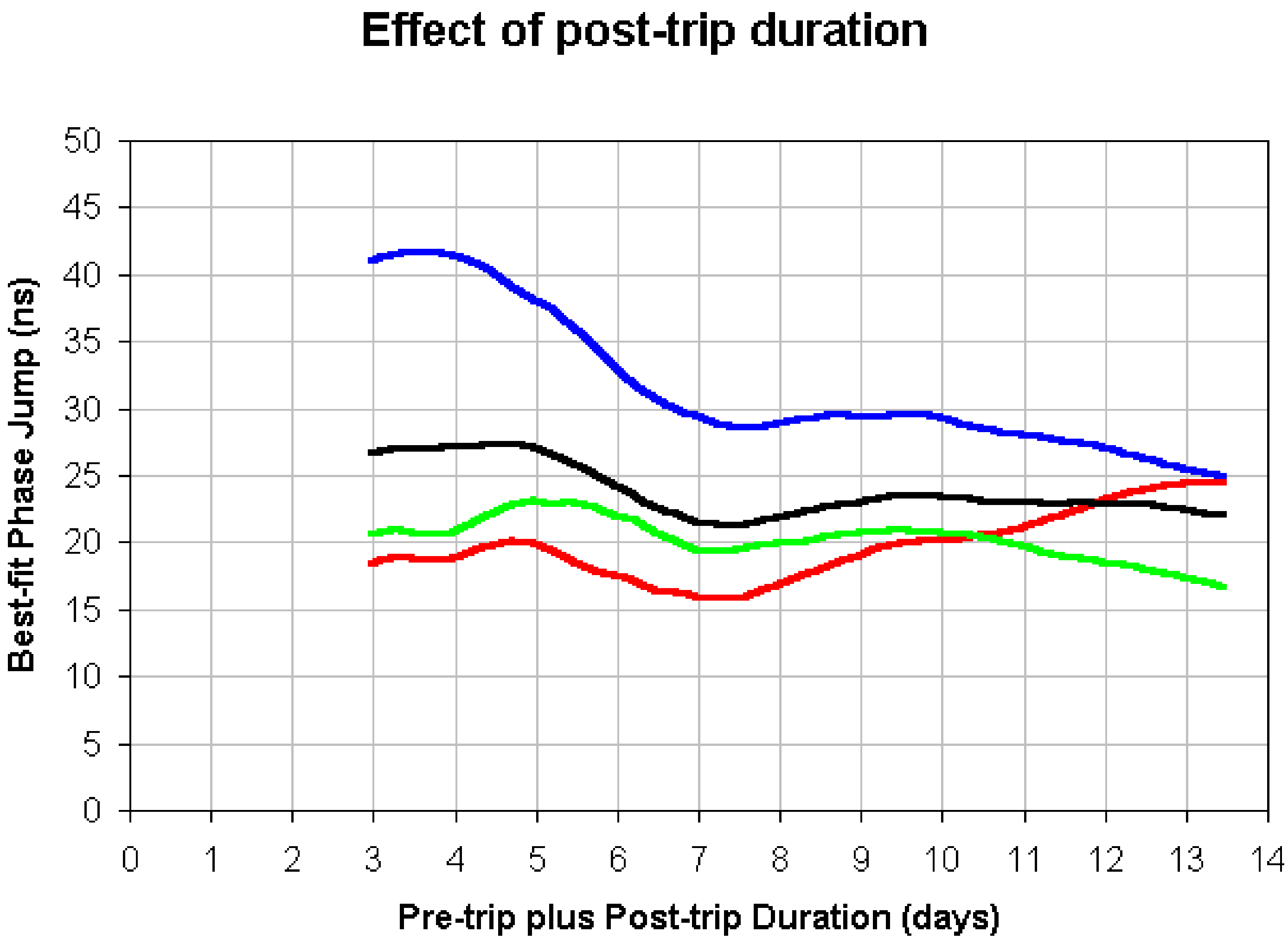
Ambiguity

- Gap in time is clear, but
- Precise magnitude depends on both pre- post-trip rate precision, which is
- Influenced by frequency averaging time
  - 1 hour? 1 day? 2 days? 7 days?
- Might be a bit subjective, but
- All methods seem to agree to a few ns

Ambiguity

- 3+3?
- 3+5?
- 3+10?
- 1+1?
- 3+3?



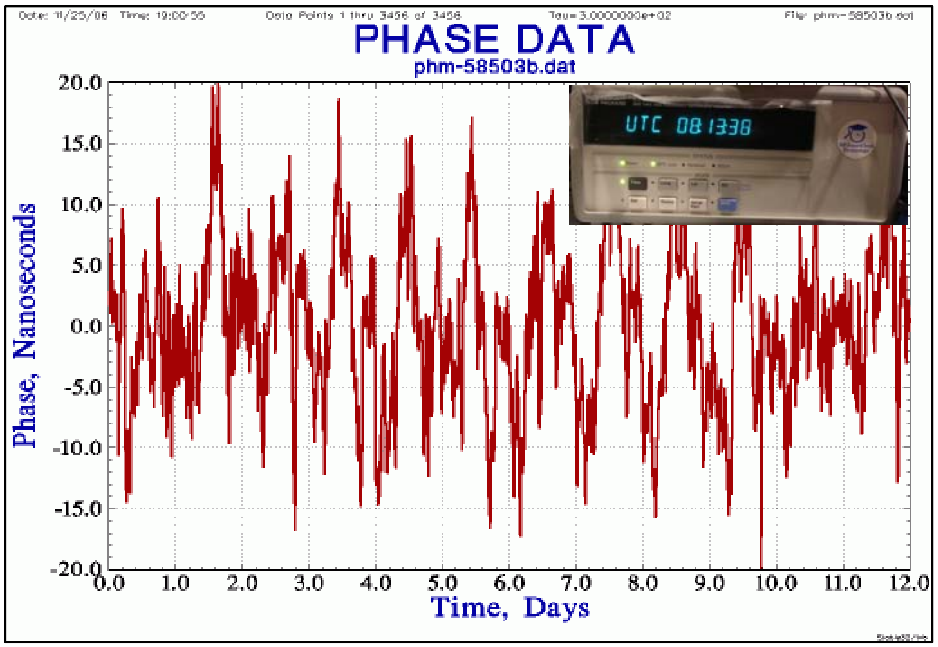


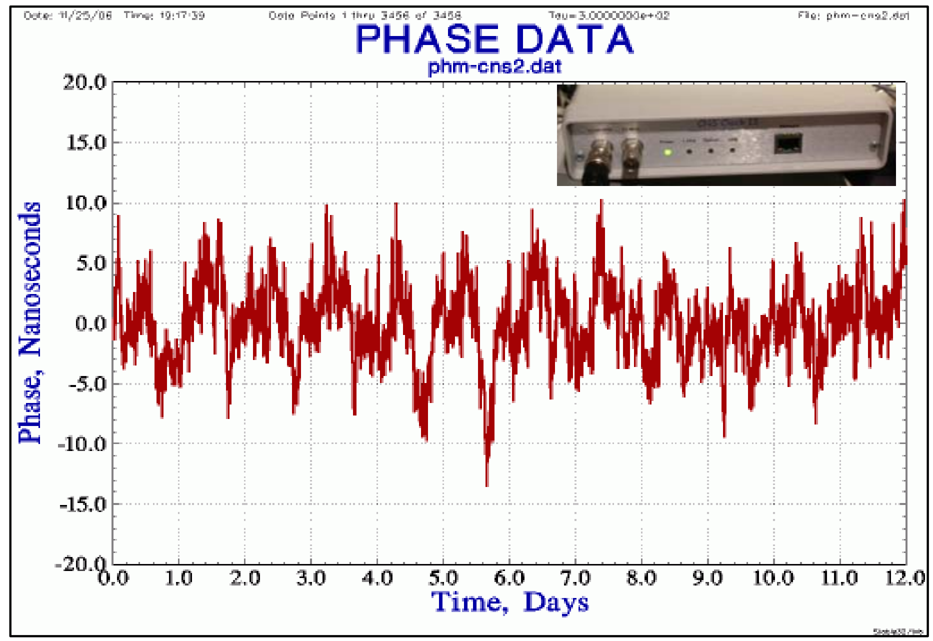
Chapter 10

- Base & portable clock performance

Base Clock

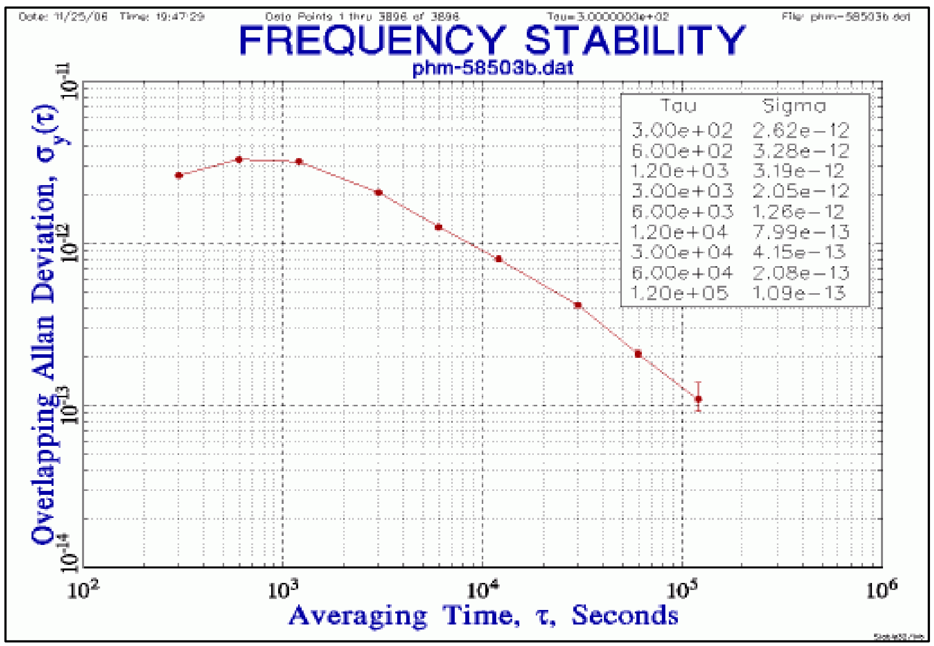
- House ref is passive H-maser
- Ref vs. GPS; phase (12-days)

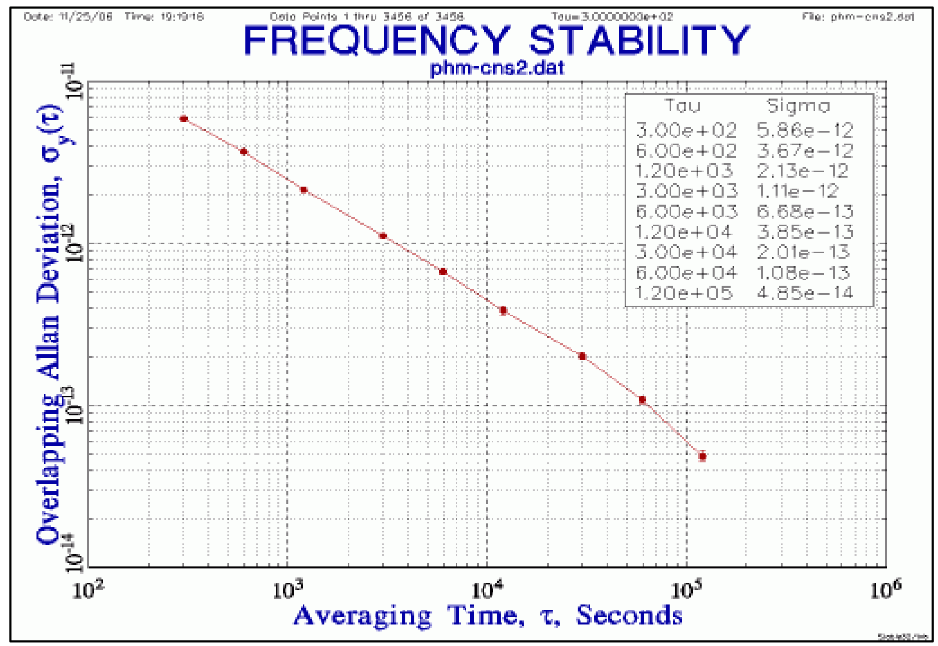




Base Clock

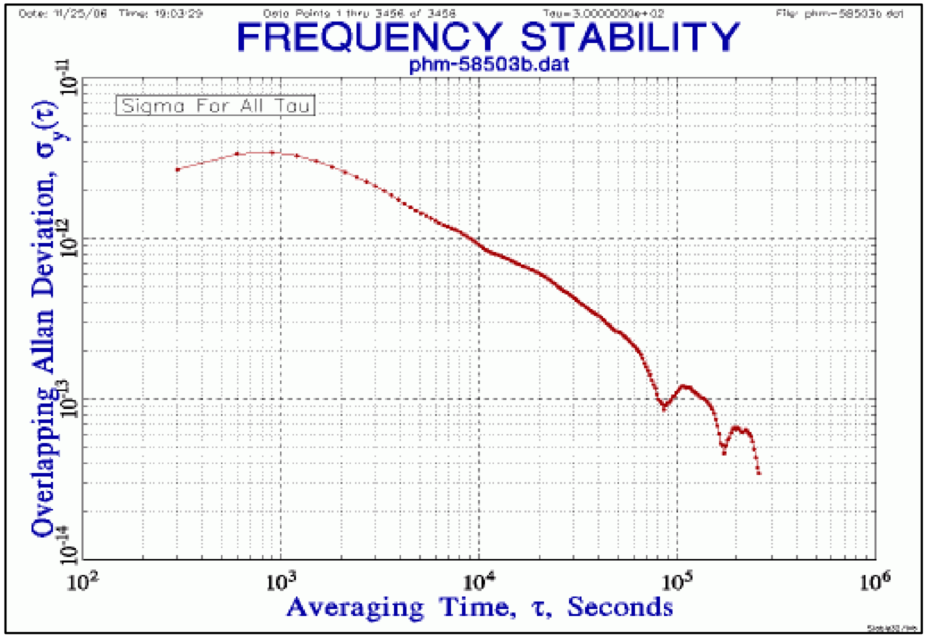
- Ref vs. GPS, ADEV (decade)
- Near/below 10^{-13} at 1 d

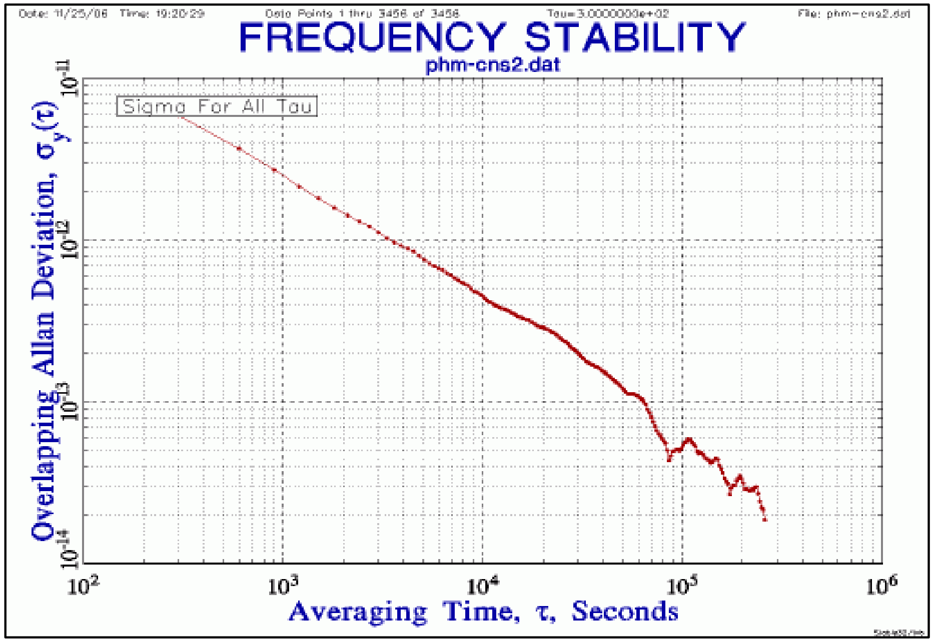




Base Clock

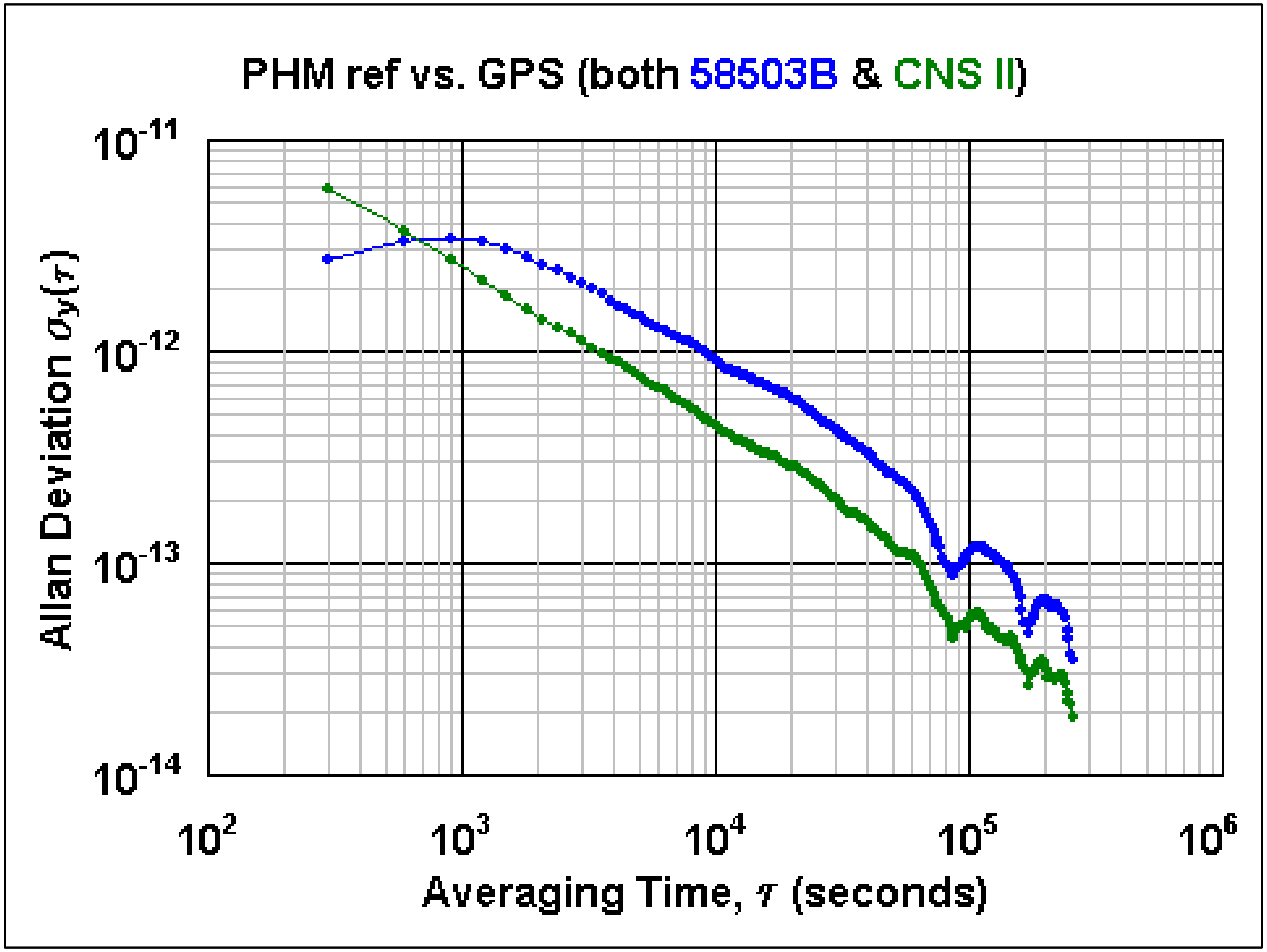
- Ref vs. GPS, ADEV (*many tau*)
- Note 1 d, 2 d better than 1.5 d





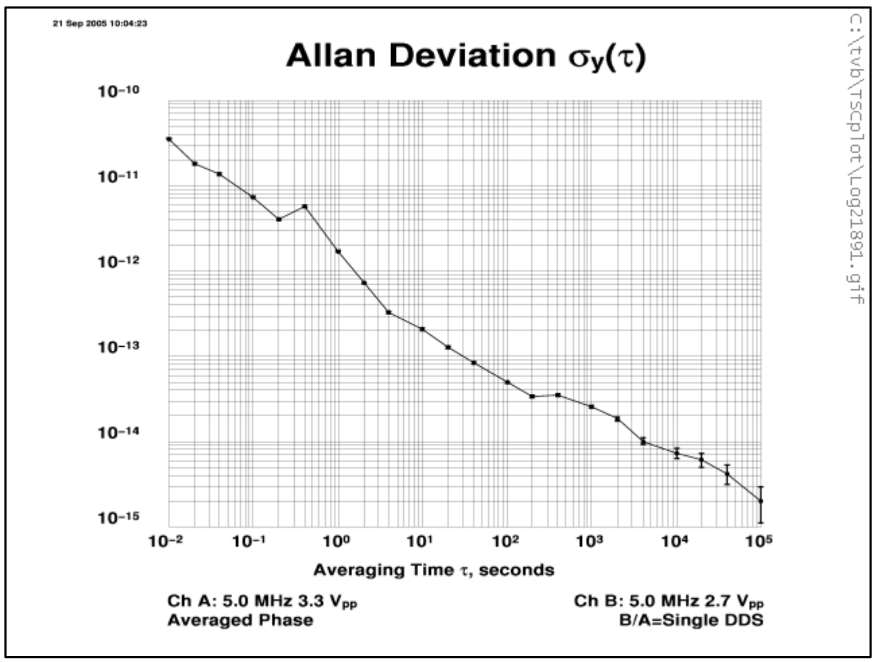
Base Clock

- Composite ADEV



Base Clock

- Passive H-maser ref vs. active H-maser
- 2×10^{-15} at 1 day means no worries



21 Sep 2005 10:08:02

Allan Deviation Table

Avg. Time (s)	Allan Deviation $\sigma_y(\tau)$	Avg. Time (s)	Allan Deviation $\sigma_y(\tau)$
0.01	$3.57\pm.11\times10^{-11}$	400	$3.54\pm.11\times10^{-14}$
0.02	$1.83\pm.057\times10^{-11}$	1000	$2.56\pm.11\times10^{-14}$
0.04	$1.371\pm.043\times10^{-11}$	2000	$1.85\pm.11\times10^{-14}$
0.1	$7.42\pm.23\times10^{-12}$	4000	$1.01\pm.084\times10^{-14}$
0.2	$4.07\pm.13\times10^{-12}$	10000	$7.3\pm.97\times10^{-15}$
0.4	$5.71\pm.18\times10^{-12}$	20000	$6.1\pm1.1\times10^{-15}$
1	$1.71\pm.054\times10^{-12}$	40000	$4.2\pm1.1\times10^{-15}$
2	$7.18\pm.22\times10^{-13}$	100000	$2.0\pm.90\times10^{-15}$
4	$3.26\pm.10\times10^{-13}$		
10	$2.08\pm.065\times10^{-13}$		
20	$1.274\pm.040\times10^{-13}$		
40	$8.34\pm.26\times10^{-14}$		
100	$4.91\pm.15\times10^{-14}$		
200	$3.41\pm.11\times10^{-14}$		

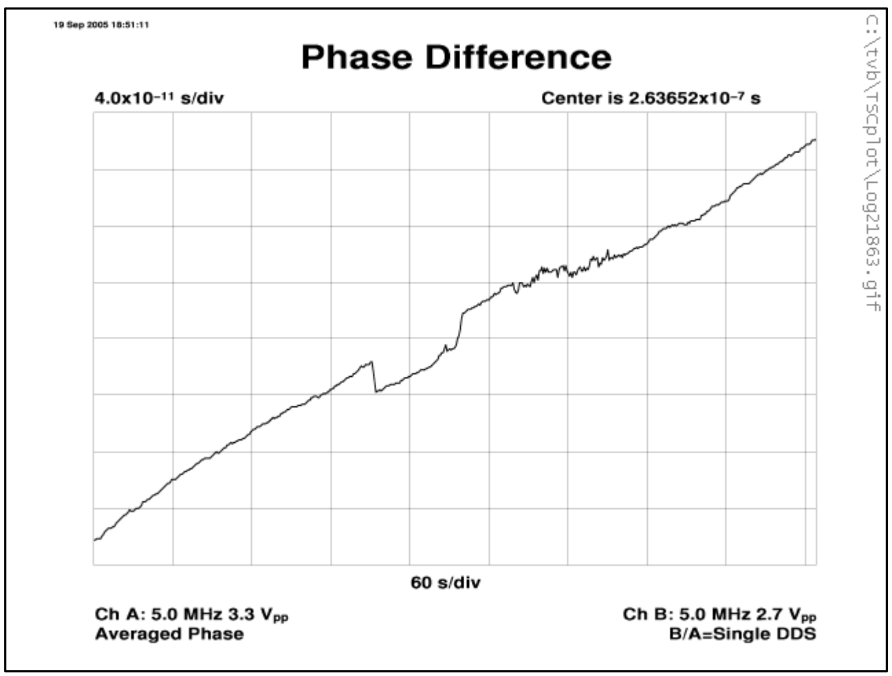
Ch A: 5.0 MHz 3.3 V_{pp}
Averaged Phase

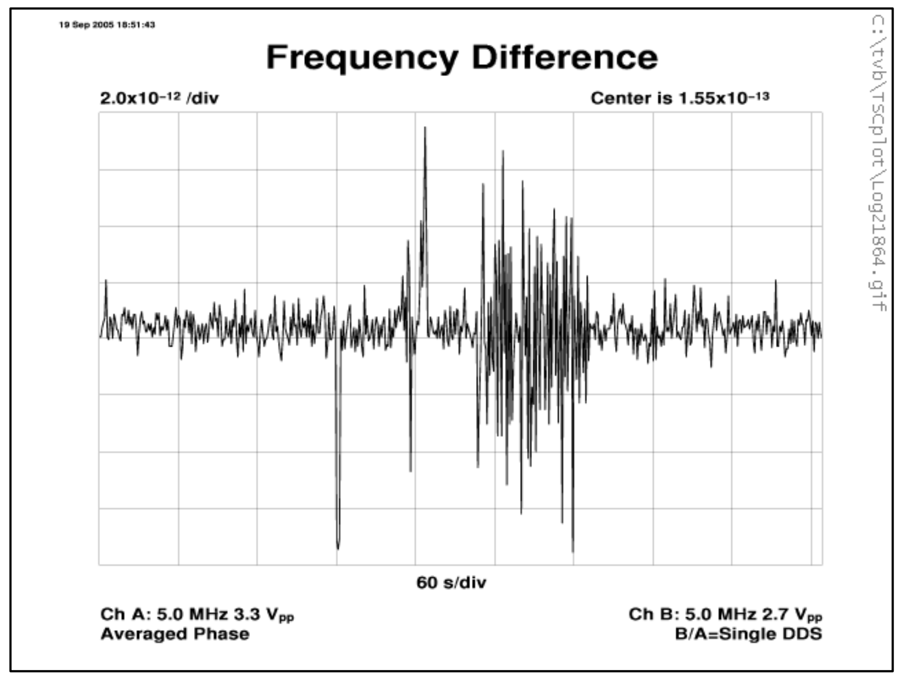
Ch B: 5.0 MHz 2.7 V_{pp}
B/A=Single DDS

C:\tvb\TscPlot\Log21892.gif

Base Clock

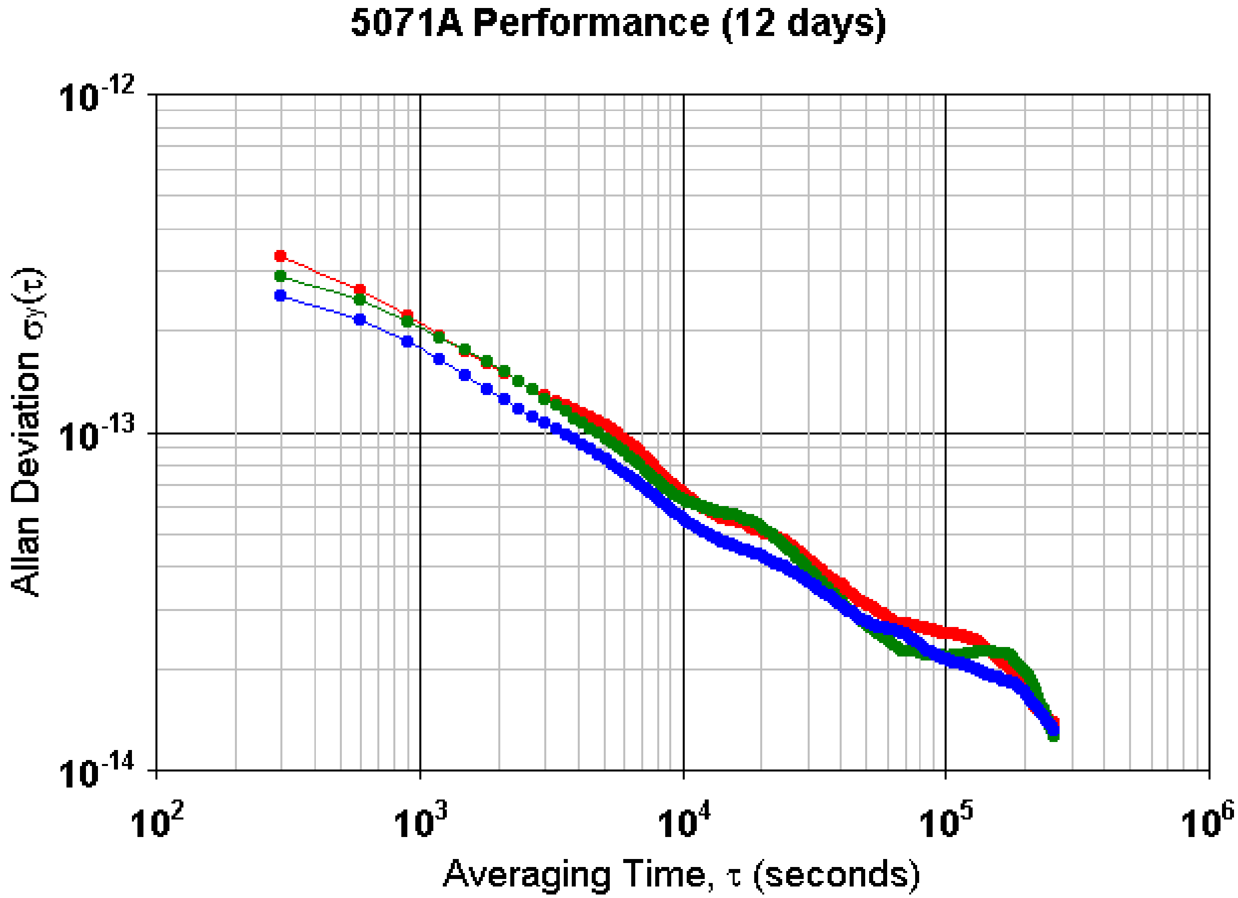
- But weird *short-term* stability
- Probably OK (20 ps jumps, 10 ps noise)





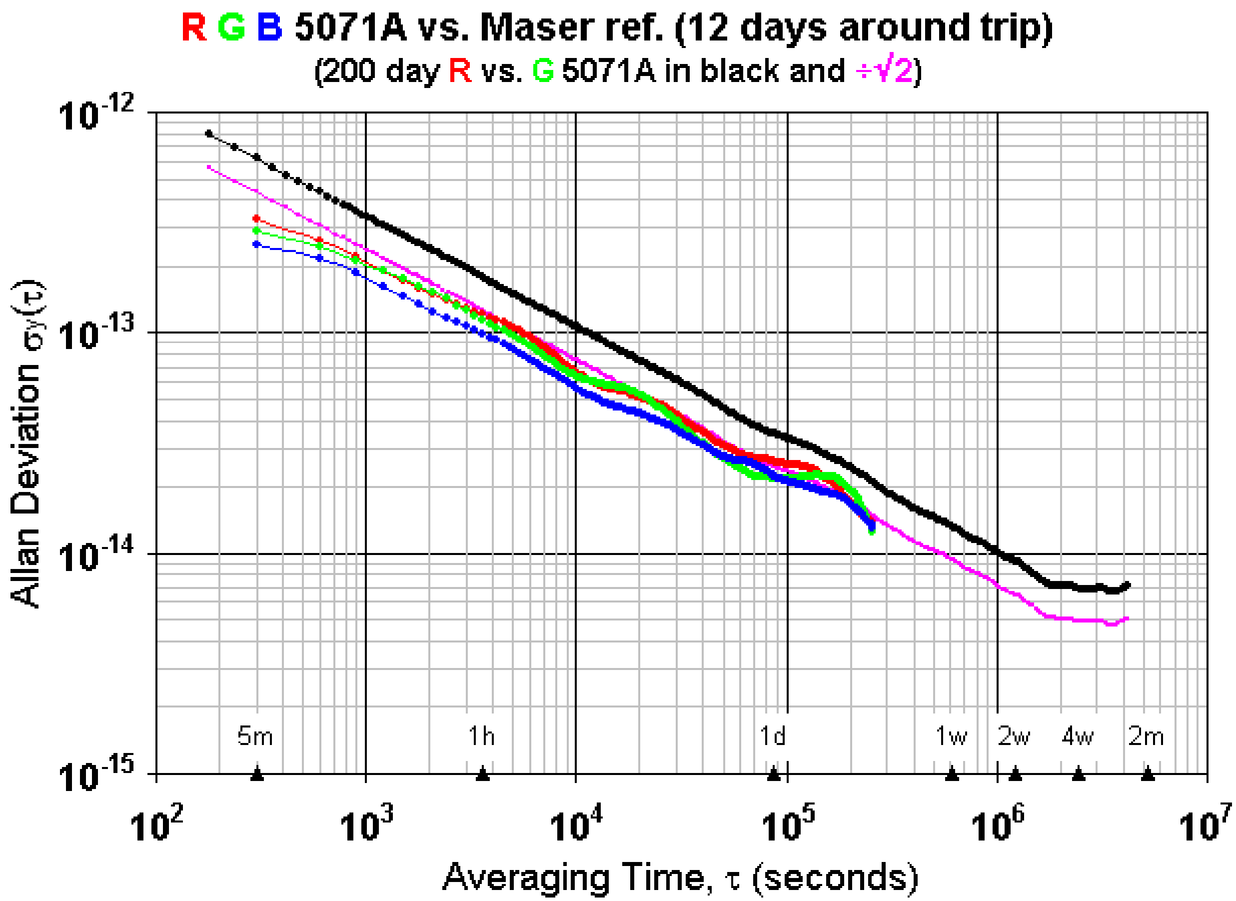
Portable Clock(s)

- ~2 days before trip + ~10 days after tip



Portable Clock(s)

- ~3 week trip would work even better

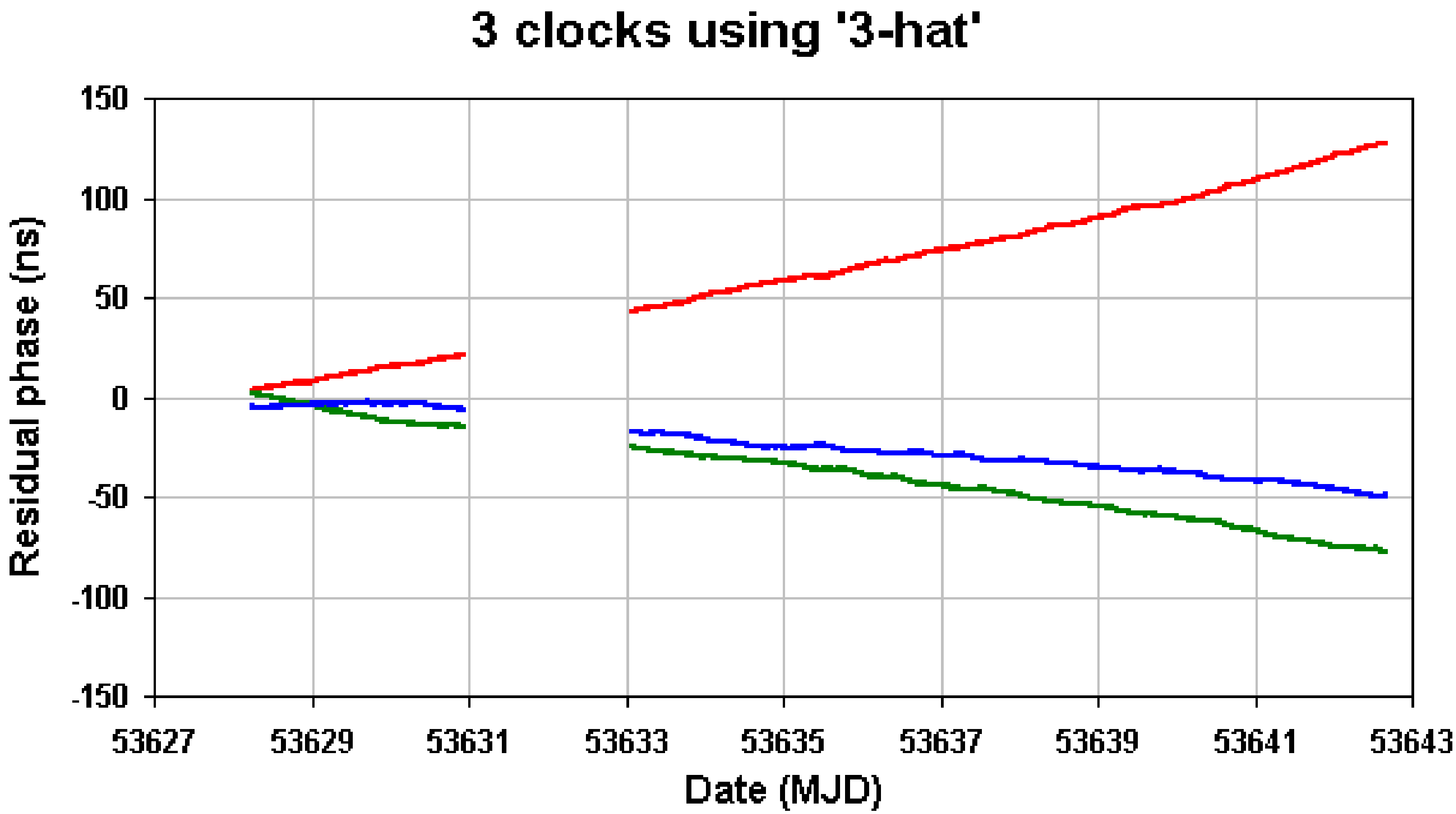


Confidence Summary

- Base clock is good to ~2×10^{-15}
- Portable clocks are good to ~2×10^{-14}
- Relativistic effect is ~1.5×10^{-13}
- GPS log is much better than 1%
- No show-stopper glitches
- So experiment accuracy is ~15%
- 15% of 22 ns is ~3 ns

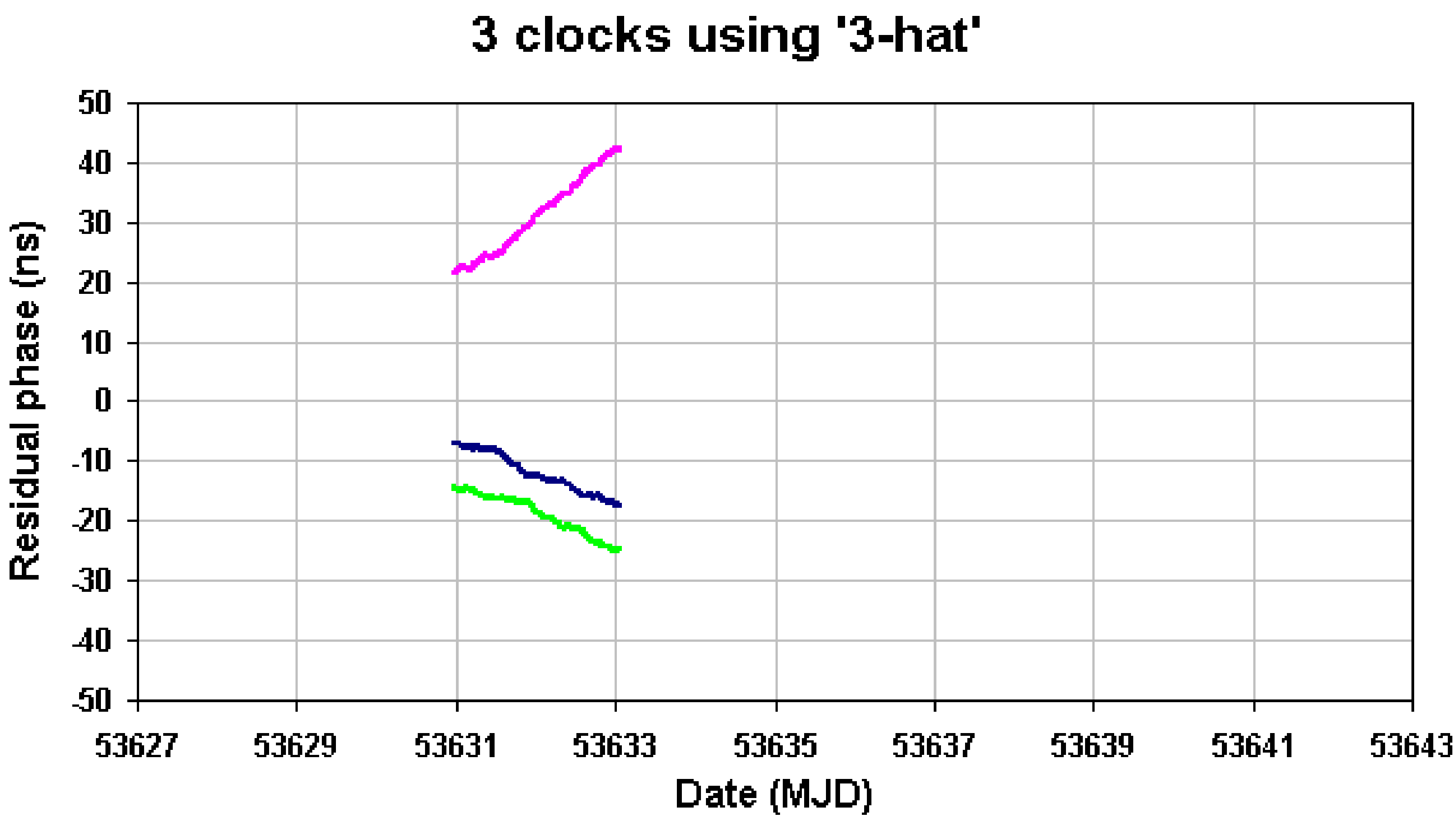
3-Hat, phase (home)

- $Cs_i - Cs_j$ via lab reference



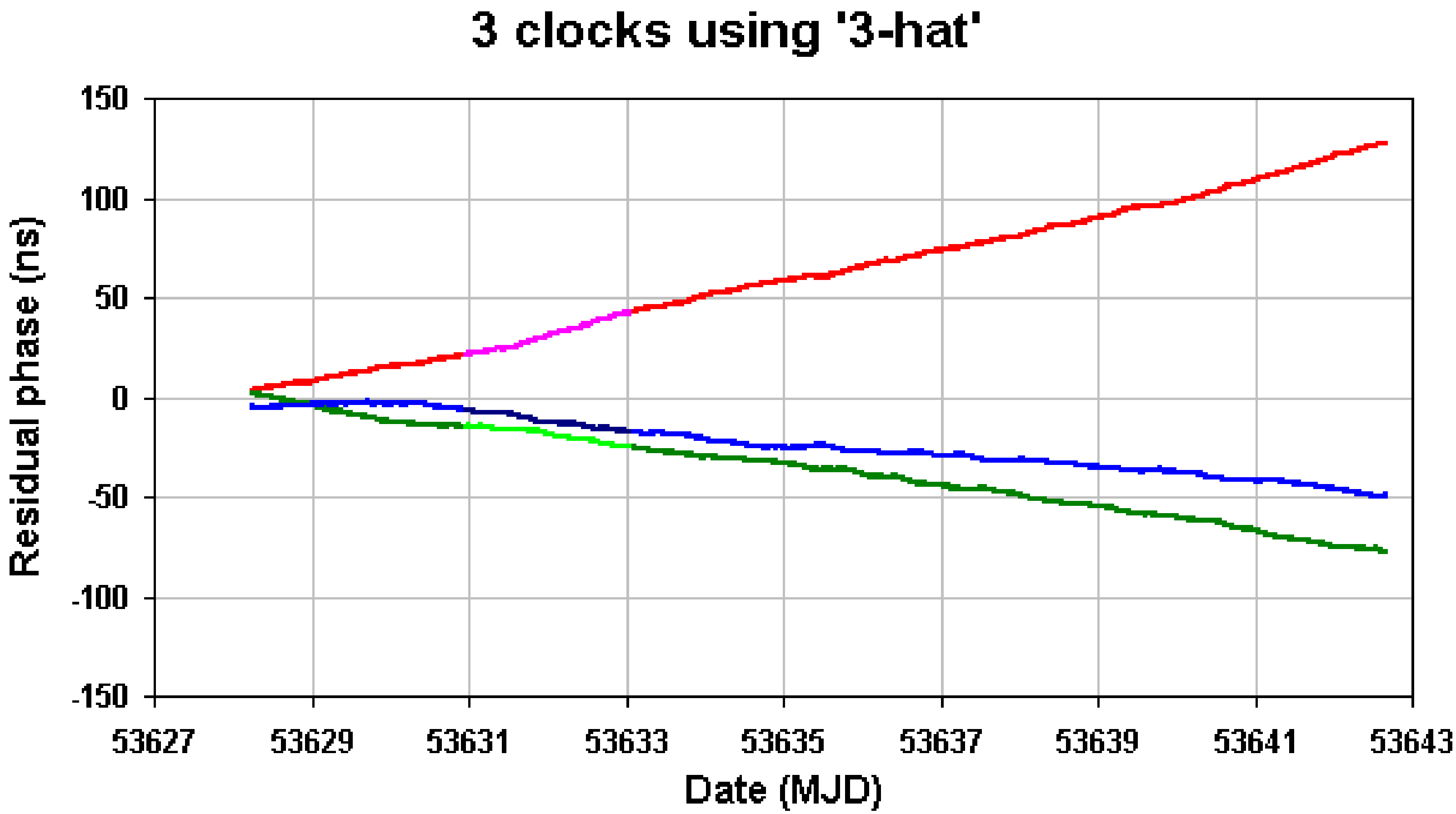
3-Hat, phase (away)

- $Cs_i - Cs_j$ via mutual-comparisons



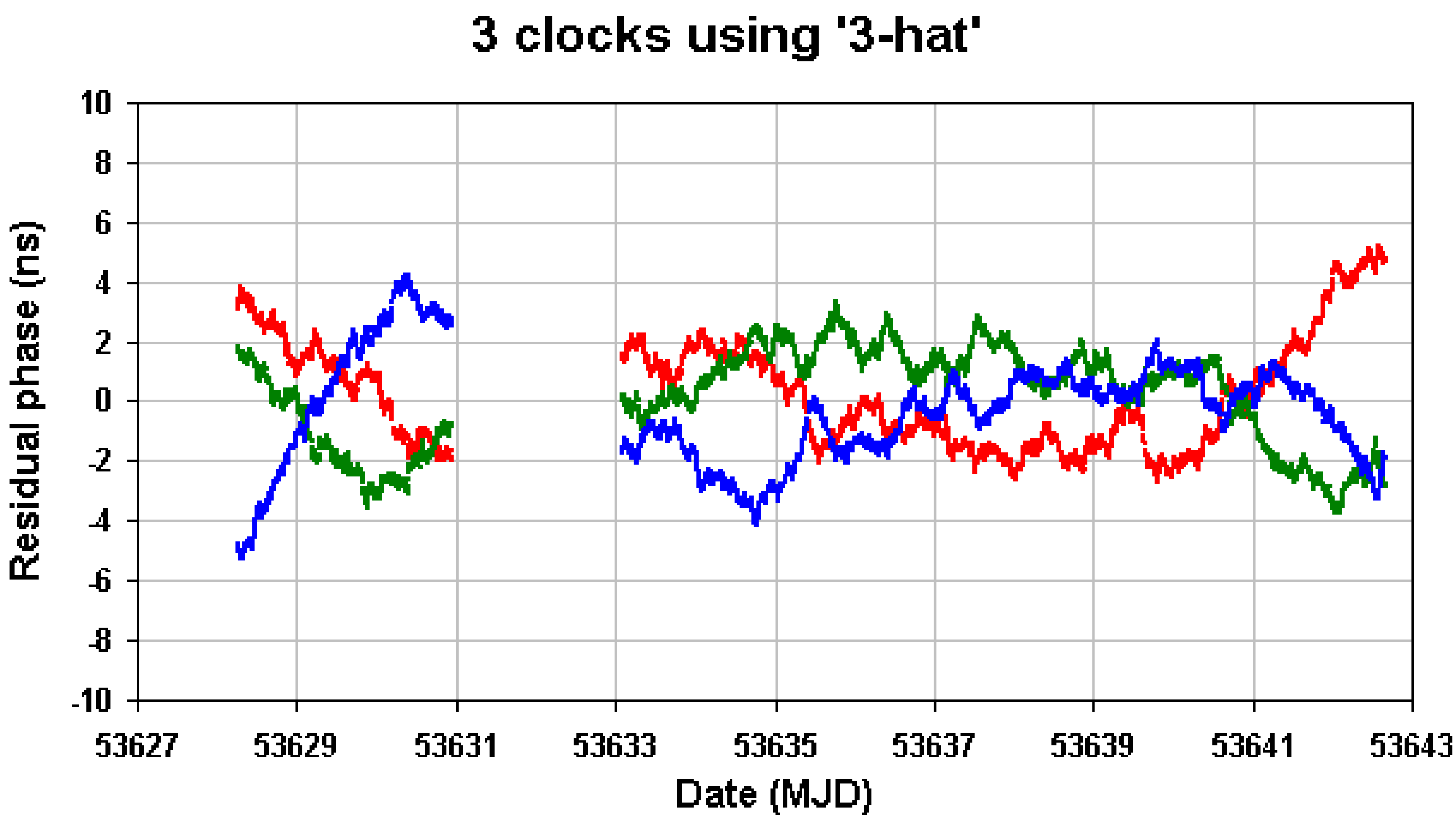
3-Hat, phase (combined)

- $Cs_i - Cs_j$ continuous



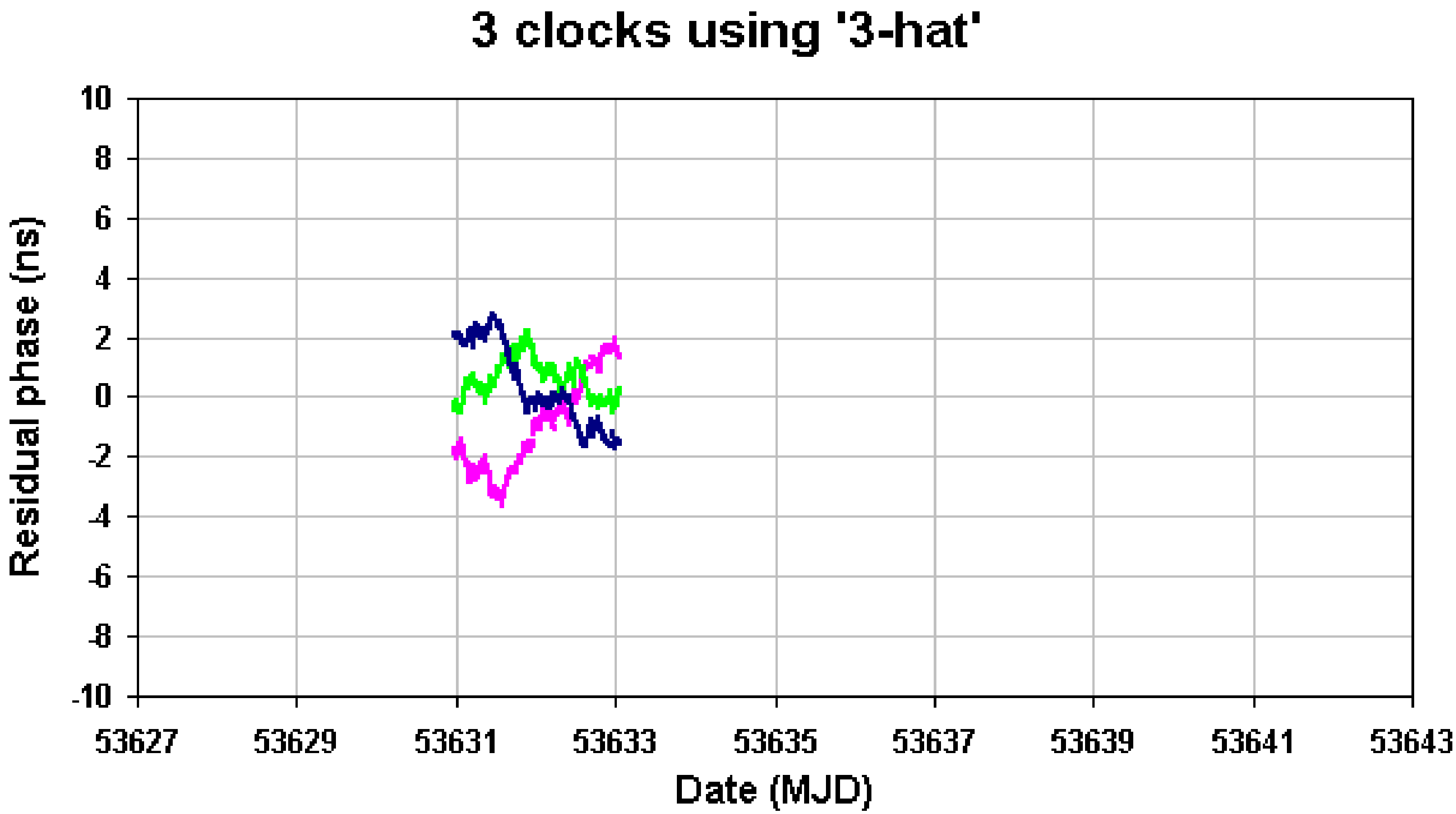
3-Hat, resid (home)

- Cs_i – Cs_j via lab reference



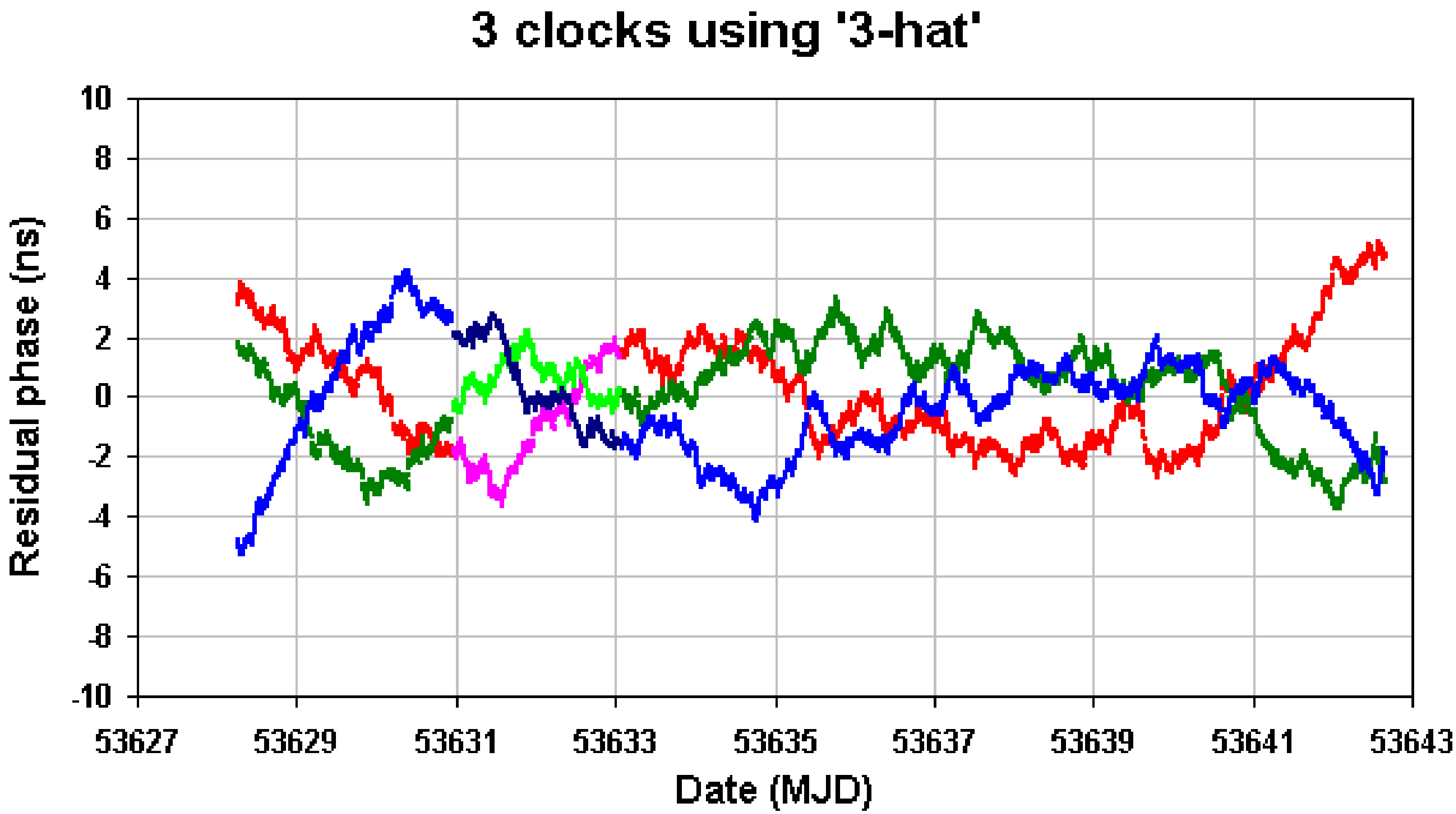
3-Hat, resid (away)

- Cs_i – Cs_j via mutual-comparisons



3-Hat, resid (combined)

- $Cs_i - Cs_j$



Final Graph; 3+2+3 days

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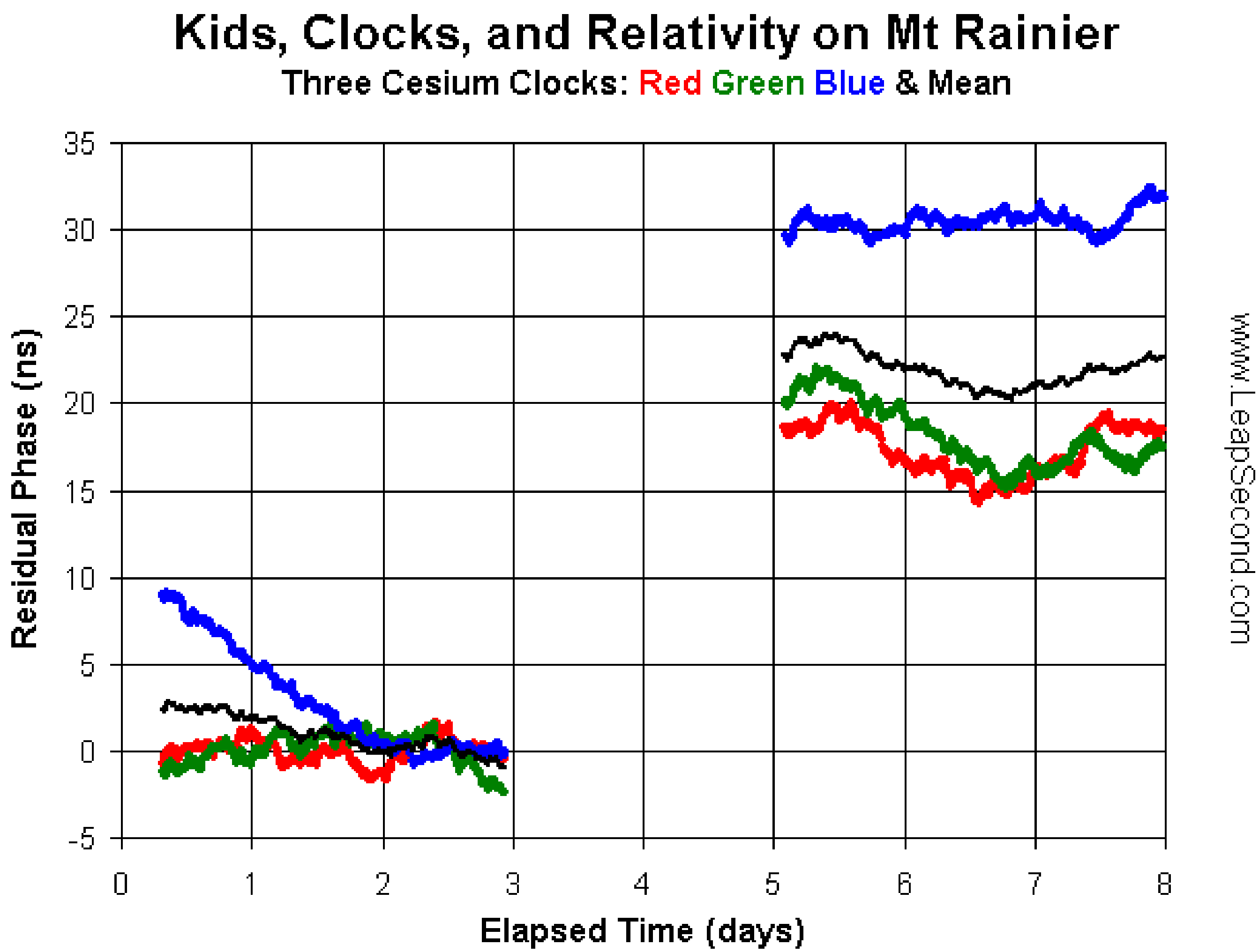
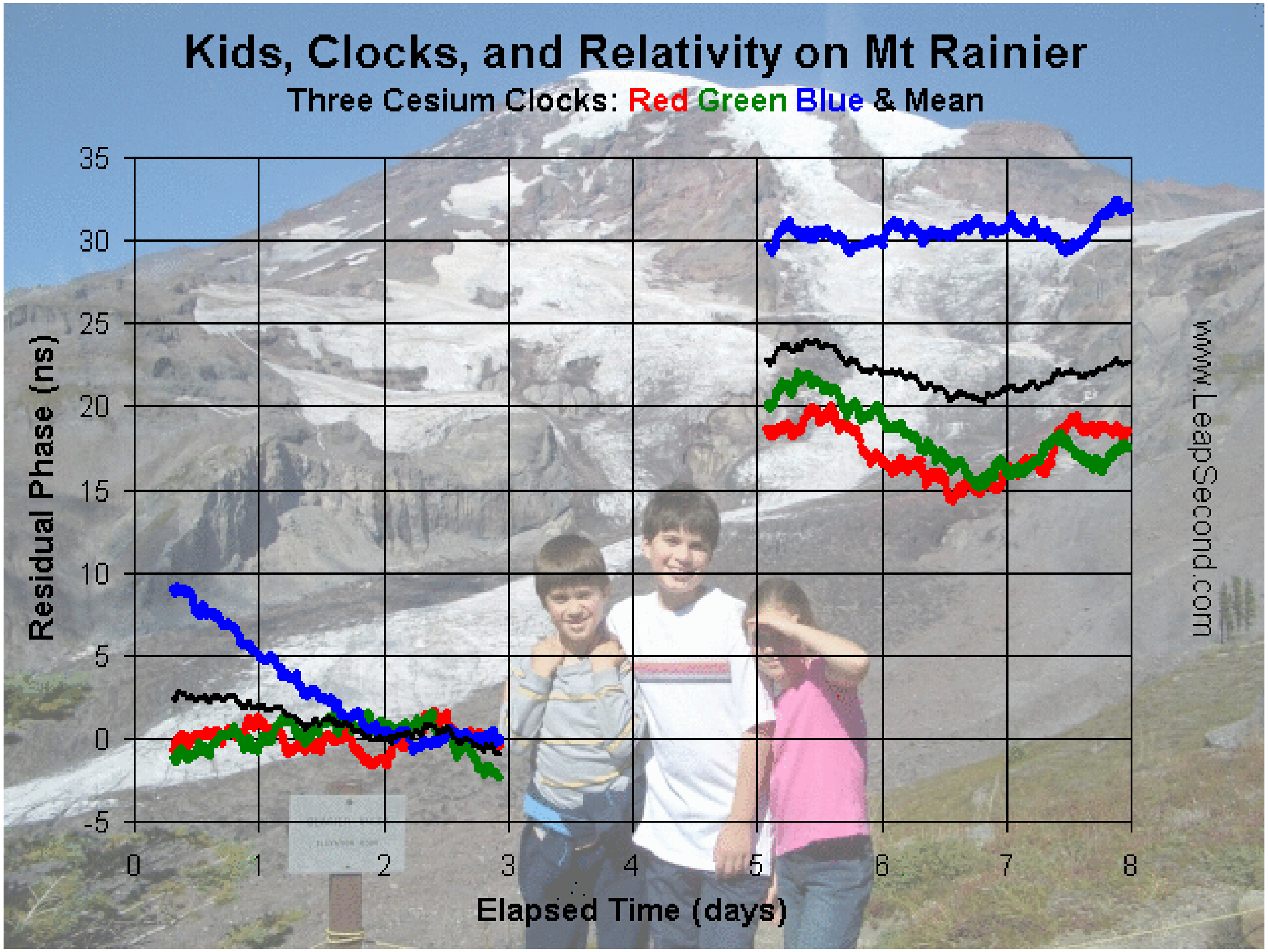


Final Graph; +kids +mountain

- 

Chapter 11

- Conclusion

What went wrong

- Difficult physical clock transfers
- Direct morning sunlight temperature
- Unexpected out-of-gas event, OK
- Hotel turned minivan off, OK
- GPS battery ran out, OK
- One TIC was 53131A (500 ps vs. 150 ps)
- Minivan A/C has poor algorithm
- Parking lot hassles, OK
- More exhausting than expected

What went right

- No loss of clock
- No loss of any data
- Relativity effects obvious and stunning
- Three clocks was good idea
- Mean closer than hoped for
- Standard deviation wider than expected
- Good input for another run
- Not bad for a first try
- Kids had a great time!

Conclusions

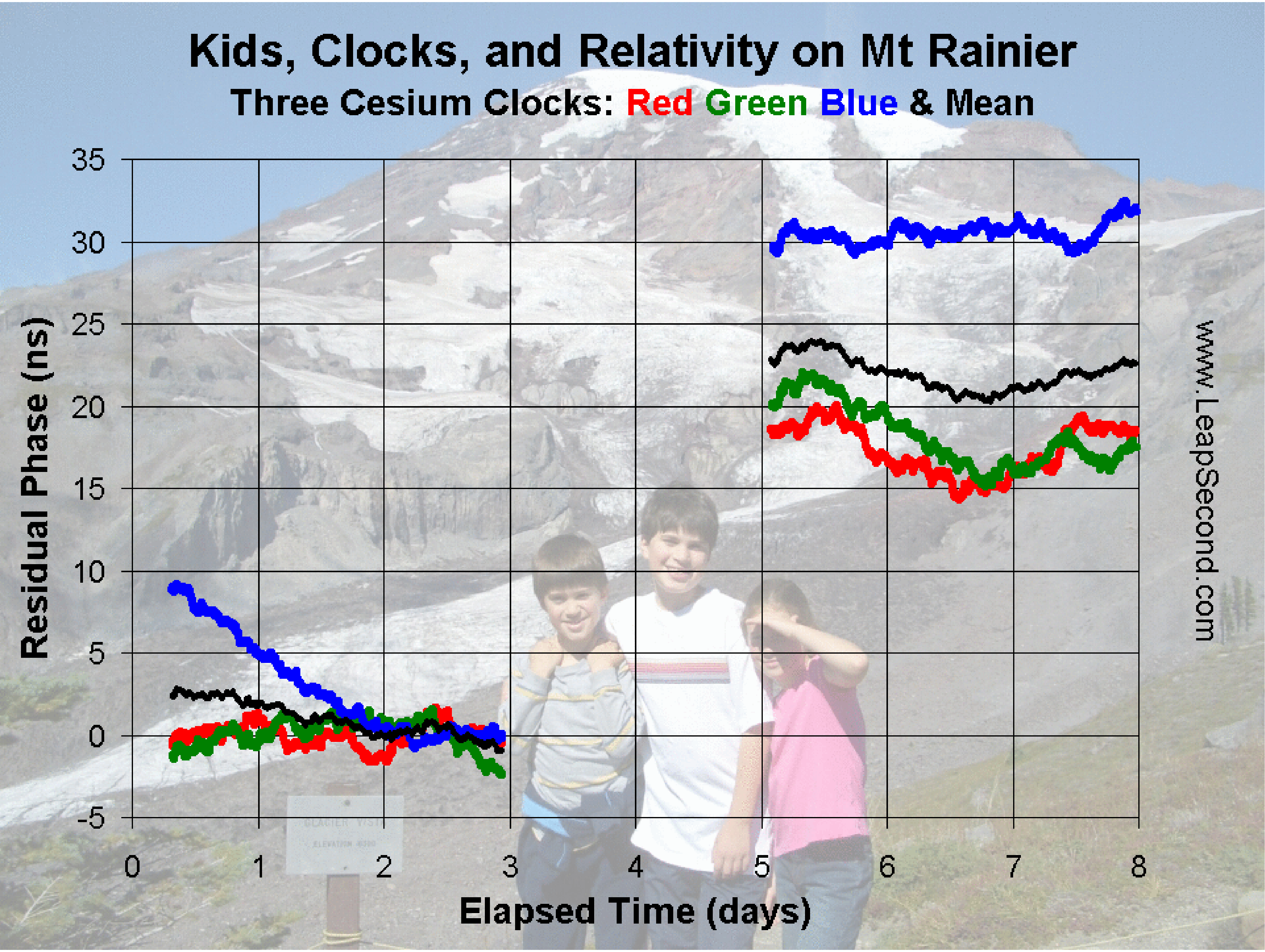
- Experiment worked rather well
- We now believe what we know
- Fun technical challenge
- Echo of historical experiments
- Useful precise time teaching example
- A fitting 2005 PTTI celebration
- Came back much* older and wiser
- Relativity is now “child’s play”
- Best 22 ns of my life!

Thank you

- For years of inspiration and support
- More info: www.LeapSecond.com
- Email: TVB@leapsecond.com
- Patience from wife & kids
- Time for questions…

GREAT^2

General Relativity
Einstein / Essen
Anniversary Test


Kids, Clocks, and Relativity on Mt Rainier
Three Cesium Clocks: Red Green Blue & Mean
Residual Phase (ns)
35
30
25
20
15
10
5
0
-5
0
1
2
3
4
5
6
7
8
Elapsed Time (days)
www.LeapSecond.com

Chapter

- Extra material

Einstein & Atomic Clocks

- Verifying GR with atomic clocks
- Did Einstein know? (he died in 1955)
- NBS Cs 1953, Essen 1955, …
- Naumann & Stroke article

Project GREAT 2007 ?

- Make minivan a lab not just a transport
- 2+ week pre-trip, 1+ week post-trip
- Single switch: ext Maser vs. int Rb ref
- Careful solar and thermal insulation
- Use park/Inn power instead of car engine
- Proper air con solution in vehicle
- Real-time plots; wireless status & alerts!
- More kids; more clocks (5?)
- Test 1970's H&K-era 5061A's for contrast
- Try direct GPS frequency measurement

Humor – cesium wristwatch

- Bill's watch…

"Atomic Bill" – first true atomic wrist watch
http://www.leapsecond.com/pages/atomic-bill/

Humor – portable cesium clock

- Tom's backpack (Project GREAT, ver 1)

"Atomic Tom" – climbing with atomic clocks
http://www.leapsecond.com/pages/atomic-tom/

Humor – 'Glimmer Twins'

- GR says high clocks run faster
- 1965: Keith gets 'high' (for 40 years)
- 2005: looks much older…

Humor – relativity

- Time flies while you're having fun
- Stay young: fly fast and low
- How to spend more time with your kids
- How to make your wife younger

Humor – make time go faster

- Charlie and the Chocolate Factory

Chapter 1

- Introduction